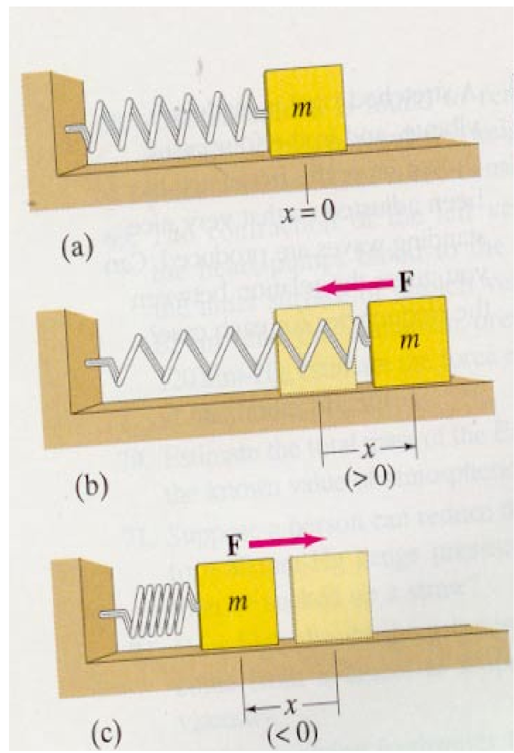
Vibrations & Waves

Many objects vibrate or oscillate….
….guitar strings, tuning forks, balance wheel of an old watch, pendulum, piano strings, bridges, radio & TV sets, atoms within a molecule and atoms within a crystal, ocean waves, earthquake waves (seismic), etc.

Spiders and sharks detect prey by vibrations on their webs and in the water respectively.

11.1 SIMPLE HARMONIC MOTION

- **Simple harmonic motion**—When a vibration or an oscillation repeats itself, back and forth [to & fro] over the same path
  - Simplest is an object oscillating on the end of a coiled spring
  - Ignore the masses of the spring & ignore friction
  - When no force is exerted by the spring, its natural length is the equilibrium position (x = 0)



- **Restoring force**—When the mass moves from equilibrium position, a force is exerted and acts in the direction of returning the mass to the equilibrium position
  - **Hooke's law**: $F = -kx$ [remember?!]
  - Valid anytime the coils don't touch or stretch or you do not exceed the elastic limit of the spring!
  - the "-" indicates that the F acts in the direction opposite to the displacement.

- Want to stretch the spring a distance x ?
  - F = +kx
  - The stiffer the spring (more stiff?), the greater the value of k.
  - NOTE that THE FORCE INVOLVED is not a constant, but varies with position. Therefore, THE ACCELERATION of M IS not a constant either! Constant force equations do not apply! Neither to constant acceleration equations!

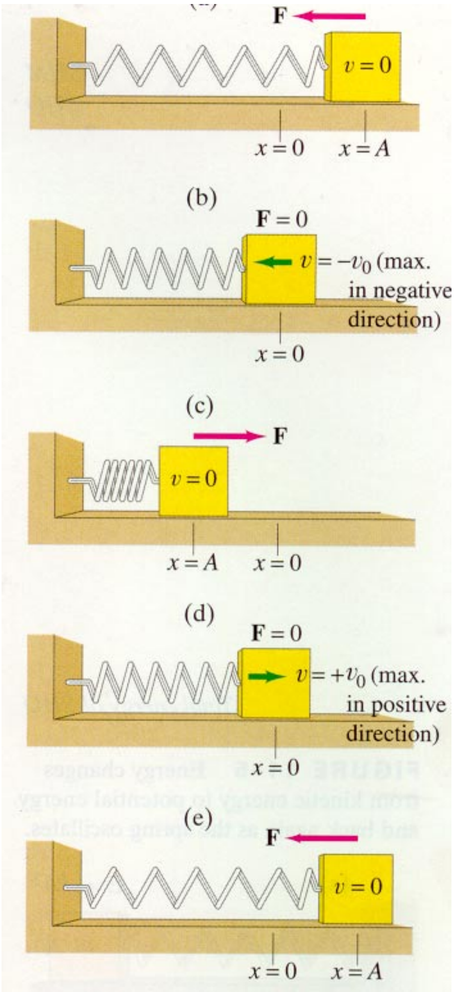


FIGURE 11–2 Force on, and velocity of, mass at different positions of its oscillation.

a) Stretch the spring a distance x = A and release.
   - m accelerates as F pulls it to the equil. positon (x = 0)
   - m passes x = 0 with considerable speed

<u>At x = 0, F decreases to zero, BUT speed is at its maximum</u>

b) m moves back in the opposite direction until…..

c) F acts to slow m as it moves to the left and <u>m stops momentarily</u> @ x = -A

d) x = A again

e) repeats motion symetrically between x = A and x = -A

We need to define a few terms:

<u>**displacement**</u>—distance, x, from equilibrium point at any moment

<u>**amplitude**</u>—maximum displacement

<u>**cycle**</u>—complete to and fro; x = A to x = -A

<u>**period**</u>—T—time to complete one cycle

<u>frequency</u>—f—number of cycles per second

<u>Hertz</u>—unit of frequency; 1 Hertz = 1 cycle/ second; 1 Hz = 1 sec $^{-1}$

NOTE THAT **T = 1/f** **and** **f = 1/T**

- What if spring is vertical rather than horizontal?

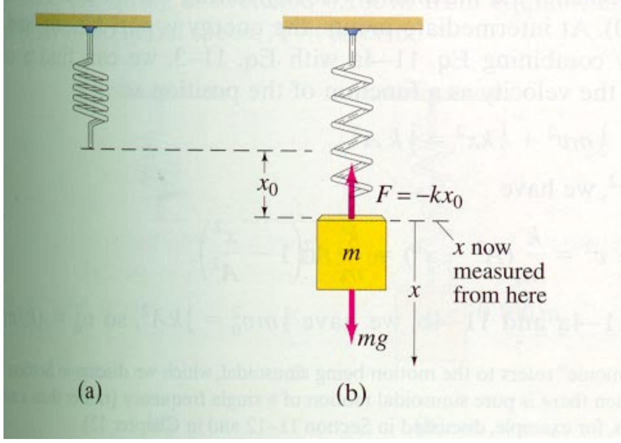


FIGURE 11–3 (a) Free spring, hung vertically. (b) Mass m attached to spring in new equilibrium position, which occurs when $\Sigma F = 0 = mg - kx_0$.

Due to gravity, the length of the vertical spring @ equilibrium will be ***<u>longer</u>*** that when horizontal.

The spring is in equilibrium when…

$\Sigma F = 0 = kx_o - mg$

so the spring stretches and "extra" amount x_o = mg/k to be in equilibrium.

Just measure x from this new position and Hooke's law applies! Set x_o as a matter of convenience when working with vertical springs.

Example 11.1

When a family of four people with a total mass of 200 kg step into their 1200 kg car. the car's springs compress 3.0 cm.

a) What is the spring constant of the car's springs assuming they act as a single spring?

b) How far will the car lower if loaded with 300 kg?

SHM—Simple Harmonic Motion is exhibited by any vibrating system for which $F_r \propto -kx$
"Simple" means single frequency and "Harmonic" means sinusoidal

SHO—Simple Harmonic Oscillator; it's what we call the above system that is exhibiting SHM.

11.2 ENERGY IN THE SHO

Since the force is NOT constant—the energy approach is much easier!

To strecth/compress a spring, Work must be done & E is stored (PE).

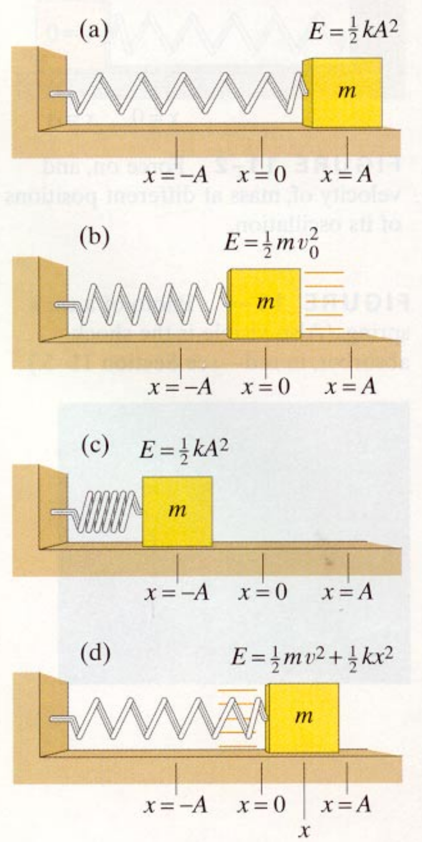
Recall that for a spring… $PE = \frac{1}{2} kx^2$

For a mass and spring system the total mechanical E = PE + KE or….

$$E = \tfrac{1}{2} kx^2 + \tfrac{1}{2} mv^2$$

ENERGY REMAINS CONSTANT IN THE ABSENCE OF FRICTION

FIGURE 11–5 Energy changes from kinetic energy to potential energy and back again as the spring oscillates.



a) At x = A and x = -A (the two extreme points) all E is PE <u>AND</u> m stops to change direction ∴ v = 0, so …..

$$E = \tfrac{1}{2} m (0)^2 + \tfrac{1}{2} kA^2 = \tfrac{1}{2} kA^2$$

This means that the total mechanical energy of a SHO is $\propto A^2$ [the square of the amplitude]

b) At the equilibrium point, x = 0 and all E is KE

$$E = \tfrac{1}{2} mv_o^2 + \tfrac{1}{2} k (0)^2 = \tfrac{1}{2} mv_o^2$$

where v_o represents the maximum v [which does occur at the equil. point, x = 0]. At intermediate points, the energy is part KE and part PE so we combine the above equation with the E = PE + KE equation and get a useful **equation for velocity as a function of position**….

$$\tfrac{1}{2} mv^2 + \tfrac{1}{2} kx^2 = \tfrac{1}{2} kA^2$$

solving for v^2… $v^2 = \frac{k}{m}(A^2 - x^2) = \frac{k}{m} A^2\left(1 - \frac{x^2}{A^2}\right)$

AND since $\tfrac{1}{2} mv_o^2 = \tfrac{1}{2} kA^2$, $v_o^2 = (k/m)A^2$ we can insert it above and take the square root…..

$$v = \pm v_o \sqrt{\left(1 - \frac{x^2}{A^2}\right)}$$

This gives v at a position x; the magnitude of velocity depends ONLY on the magnitude of x

Example 11.2

Suppose the spring in figure 11.5 is stretched twice as far (to x = 2A). What happens to the energy of the system?

What happens to the maximum velocity?

What happens to the maximum acceleration?

Example 11.3

A spring stretches 0.150 m when a 0.300 kg mass is hung from it. The spring is then stretched an additional 0.100 m from this equilibrium point, and released. Determine

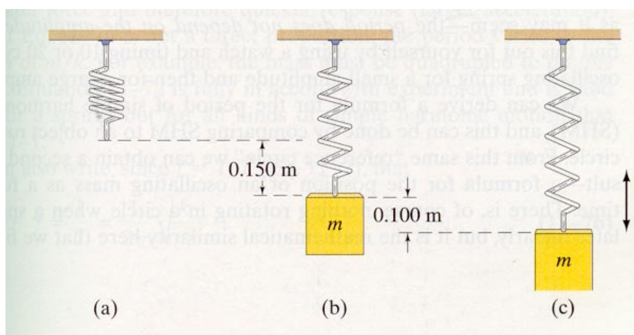


FIGURE 11–6 Examples 11–3 and 11–4.

a) the spring constant

b) the amplitude of the oscillation

c) the maximum velocity

d) the magnitude of the velocity when the mass is 0.050 m from equilibrium

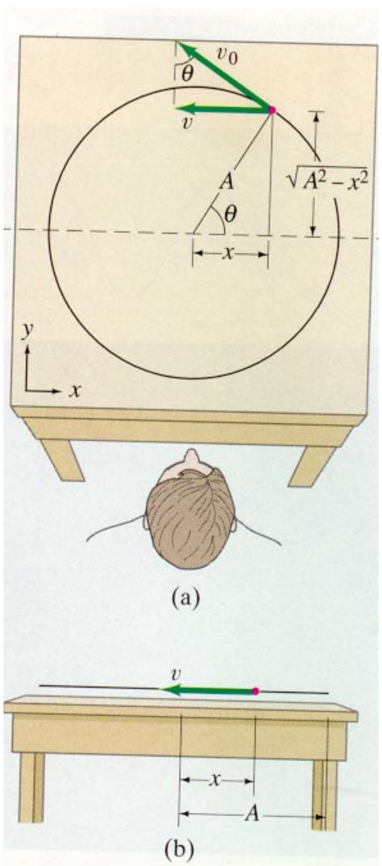
e) the magnitude of the maximum acceleration of the mass.

Example 11.4

For the simple harmonic oscillator of the previous example, determine

a) the total E

b) The KE and PE at half amplitude ($x = \pm A/2$)



GURE 11–7 Analysis of
ıple harmonic motion as a side
w (b) of circular motion (a).

11.3 THE PERIOD AND SINUSOIDAL NATURE OF SHM

The period, T, of a SHO depends on m & k but ***NOT*** on [strange as it may seem] amplitude.

Compare SHM to an object rotating in a circle—we'll call it the reference circle. We can derive a formula for the period of SHM using this method. We also obtain a second useful result, a formula for the position of an oscillating mass as a function of time.

There is, of course, nothing rotating in a circle when a spring oscillates linearly, but it is the mathematical similarity here that we find useful [and moderately entertaining!]. It may take you a couple of tries to follow this—be patient, it will make sense!

Consider a mass revolving counter clockwise in a circle of radius A with constant speed v_o, on top of a table.

Look from above the table and the motion is a circle in the x-y plane, BUT look from the edge of the table, eye-level, and all you see is one-dimensional motion corresponding precisely to SHM. We are interested in projection of circular motion onto the x-axis and the magnitude of the component of v.

The two triangles involving θ are similar

$$\frac{v}{v_o} = \frac{(A^2 - x^2)^{1/2}}{A} \quad \text{OR} \quad v = \pm v_o \sqrt{\left(1 - \frac{x^2}{A^2}\right)}$$ which we've seen lately!

Thus, the projection on the x-axis of an object revolving in a circle has the same motion as a mass at the end of a spring.

Now….T = one revolution

$$v_o = \frac{2\pi A}{T} = 2\pi A f$$

solve for the period, T

$T = \dfrac{2\pi A}{v_o}$ and $\frac{1}{2} kA^2 = \frac{1}{2} m v_o^2$ so $\dfrac{A}{v_o} = (m/k)^{1/2}$

Thus

$$T = 2\pi \sqrt{\frac{m}{k}}$$

The period depends on m & k , NOT amplitude.

- greater m $\leftrightarrow$ longer T
- greater k (stiffer) $\leftrightarrow$ shorter T
- mass must quadruple to double T since it is a squared relationship

Since T = 1/f we can also write

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Exercise 11.5

What are the period and frequency of the car in example 11.1 after hitting a bump? Assume the shock absorbers are poor, so the car really oscillates up and down.

Example 11.6

A small insect of mass 0.30 g is caught in a spiderweb of negligible mass. The web vibrates predominantly with a frequency of 15 Hz.

a) Estimate the value of the spring constant for the web.

b) At what frequency would you expect the web to vibrate if an insect of mass 0.10 g were trapped?

We can also use the reference circle to find the *position of a mass undergoing SHM as a function of time.* From figure 11.7 we see that cos θ = x/A, so the projection of the ball's position on the x axis is

$$x = A \cos \theta$$

Since the mass is rotating with angular velocity ω, we can write θ = ωt, where θ is in radians. Thus

$$x = A \cos \omega t$$

Furthermore, the angular velocity (specified in radians per second) can be written as ω = 2πf

$$x = A \cos 2\pi f t, \text{ or in terms of period T:}$$

Position as a function of time $\quad x = A \cos \left(\frac{2\pi t}{T} \right)$

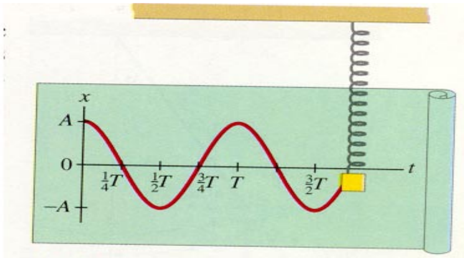


FIGURE 11–9 Sinusoidal nature of SHM as a function of time; in this case, $x = A \cos(2\pi t/T)$.

When t = T [that is after a time = to one period] we have the cosine of 2π, which is the cosine of zero. This makes sense since the motion repeats itself after a time T.

Since the cosine function varies between 1 and –1, then x varies between A and –A. Attach a pen to the vibrating mass and as shown in fig. 11.9 and a sheet of paper is moved at a steady rate beneath it, a curve will be drawn that shows SHM is sinusoidal.

We can also find velocity v as a function of time.
For the point shown, the [red] dot:

$v = v_o \sin\theta$ but the vector **v** points to the left, so $v = -v_o \sin\theta$

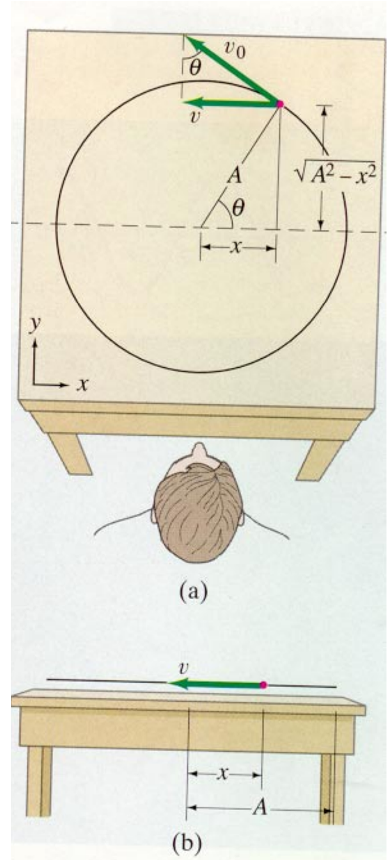
Again, setting $\theta = \omega t = 2\pi ft = 2\pi t/T$ we have

$$v = -v_o \sin 2\pi ft = -v_o \sin \frac{2\pi t}{T}$$

At t = 0 the velocity is negative (points to the left) and remains so until t = ½ T {corresponding to θ = 180° = π radians}.

Then from t = ½ T until t = T the velocity is positive.

The velocity as a function of time is plotted in figure 11.10b



GURE 11–7 Analysis of
ıple harmonic motion as a side
w (b) of circular motion (a).

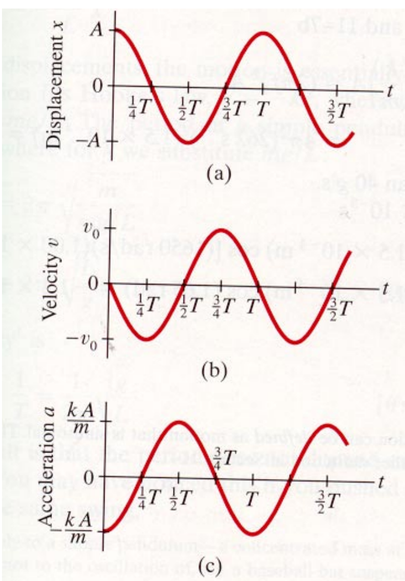


FIGURE 11–10 Graphs showing (a) displacement x as a function of time t: $x = A\cos(2\pi t/T)$; (b) velocity as a function of time: $v = -v_0 \sin(2\pi t/T)$; (c) acceleration as a function of time, $a = -(kA/m)\cos(2\pi t/T)$.

Recall that $v_o = 2\pi Af$ so…

$$v_o = 2\pi Af = A\sqrt{\frac{k}{m}}$$

The acceleration as a function of time is readily obtained from Newton's 2nd law, F = ma:

$$a = \frac{F}{m} = \frac{-kx}{m} = -\frac{kA}{m}\cos 2\pi ft = -a_o \cos 2\pi ft = -a_o \cos \frac{2\pi}{T} t$$

where the maximum acceleration is $a_o = kA/m$

This equation is plotted in figure 11.10c. Because the acceleration of a SHO is NOT constant, the equations for uniformly accelerated motion do NOT apply to SHM.

Other equations for SHM are also possible. If at t = 0 you p*ush* a mass to begin oscillations the equation would be

$$x = A \sin \frac{2\pi t}{T}$$

This has precisely the same shape as the cosine curve in figure 11.10a, it's just shifted by a ¼ cycle so that it starts out at x = 0 rather than x = A. **Both of the curves, sine and cosine, are considered sinusoidal. Thus, SHM is said to be sinusoidal because the position varies as a sinusoidal function of time.**

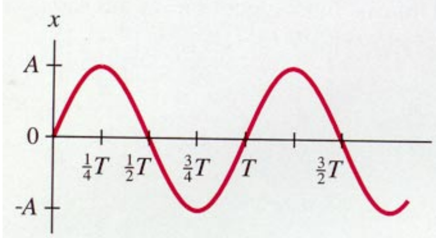


FIGURE 11–11 Sinusoidal nature of SHM as a function of time; in this case, $x = A \sin(2\pi t/T)$ because at $t = 0$ the mass is at the equilibrium position $x = 0$, but it also has (or is given) an initial speed at $t = 0$ that carries it to $x = A$ at $t = \frac{1}{4}T$.

Example 11.7

The cone of a loudspeaker vibrates in SHM at a frequency of 262 Hz (middle C). The amplitude at the center of the cone is $A = 1.5 \times 10^{-4}$ m, and at t = 0, x = A.

a) What is the equation describing the motion of the center of the cone?

b) What is the maximum velocity and maximum acceleration?

c) What is the position of the cone at t = 1.00 ms?

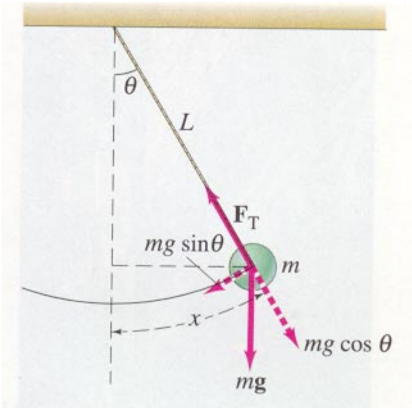


FIGURE 11–12 Simple pendulum.

THE SIMPLE PENDULUM

<u>Simple pendulum</u>—small object [bob] suspended from the end of a lightweight cord—ignore the mass of the cord relative to the bob. Swing it back and forth and it resembles simple harmonic motion—or does it? Is the restoring force ∝ to the displacement?

The displacement along the arc is given by x = Lθ. Thus, if the restoring force is ∝ to x or to θ, it's SHM.

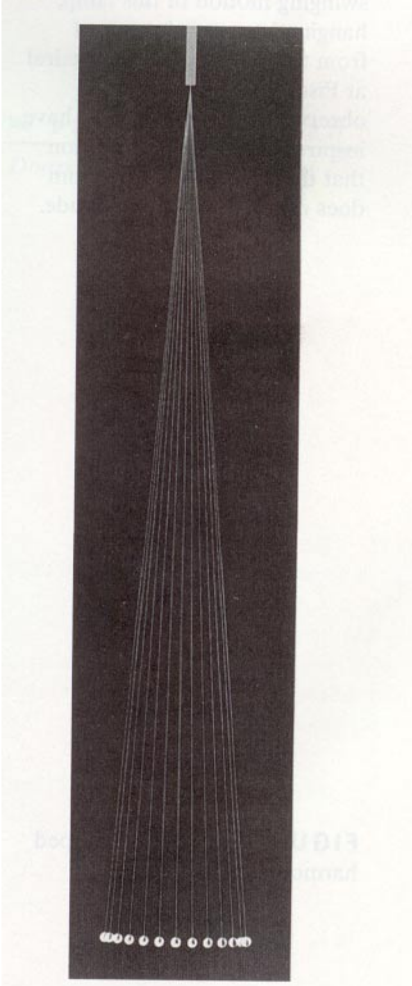
F_r = -mg sin θ

Since F is ∝ to the sine of θ and not θ, itself it's NOT SHM.

Physicists call this next part "hand waving"!

If we have a very small θ, then sin θ is very near θ when θ is in radians. If θ is 15° or less the difference between θ and sinθ is less than 1% IF done in radians.

FIGURE 11–13 Strobe-light photo of an oscillating pendulum.



So what?……
For small angles

$$F = -\,mg \sin\theta \approx -\,mg\theta$$

AND the chord is really close to x = Lθ so we can say [hand waving—fudging—near cheating here]

$$F \approx -\left(\frac{mg}{L}\right)x$$

Thus, for small dislpacements, the motion is essentially simple harmonic, since this equation fits Hooke's law, F = -kx, where the effective force constant is k = mg/L.

The period of a simple pendulum can be found using

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}}$$

When θ is small,
the period of a pendulum is: $$T = 2\pi\sqrt{\frac{L}{g}}$$

A surprising result is that the period does NOT depend on the mass of the pendulum bob! For any object having SHM, we already established that the T does NOT depend on amplitude either!

Example 11.8

a) Estimate the length of the pendulum in a grandfather clock that ticks once per second.

b) What would be the period of a clock with a 1.0 m long pendulum?

11.4 DAMPED HARMONIC MOTION

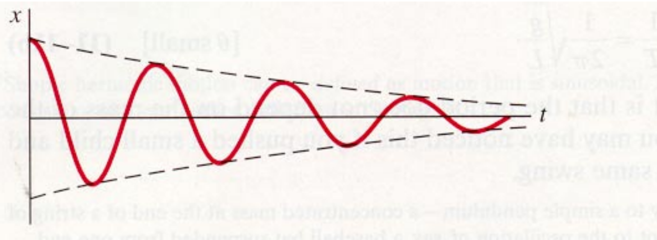


FIGURE 11–15 Damped harmonic motion.

This graph shows displacement as a function of time. The damping is due to friction [internal to the system] and air resistance [friction that is external to the system]. Ahhhh—reality!

SHM is just more math friendly! Gotta start somewhere!

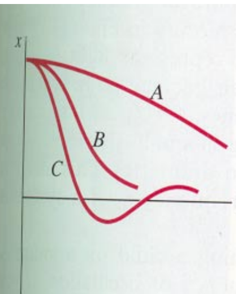


FIGURE 11–16 Graphs that represent (*A*) overdamped, (*B*) critically damped, and (*C*) underdamped oscillatory motion.

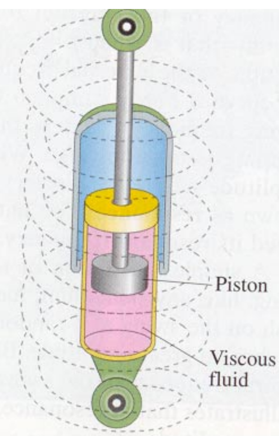


FIGURE 11–17 Automobile spring and shock absorber to provide damping so that car won't bounce up and down endlessly.

Frictional damping does alter the frequency of vibration, but the effect is small unless the damping is large. Sometimes the damping is huge and SHM is no longer recognized.

overdamped—Curve A in fig. 11.16—damping is so large it takes a LONG time to reach equil.
underdamped—Curve C—the system makes several swings before coming to rest
critical damping—Curve B—equilibrium is reached the quickest

Door closing mechanisms and shock absorbers are places where damping is used to "smooth" things out by critical damping. As they wear out, underdamping occurs and they slam or bounce you up and down as you go over a bump in a car.

11.5 FORCED VIBRATIONS; RESONANCE

When a vibrating system is set into motion, it vibrates at its natural frequency.
Forced vibration—occurs when an external foce is applied to a vibrating system that has its own particular frequency.
Natural frequency (aka resonant frequency) —denoted by f_o;

$$f_o = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- For a forced vibration, A depends on the difference between f and f_o

- A is a maximum when $f = f_o$

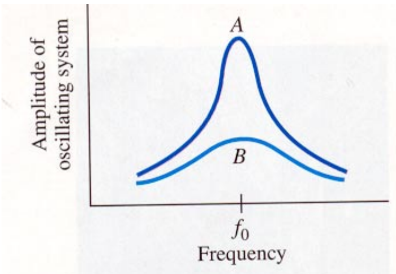
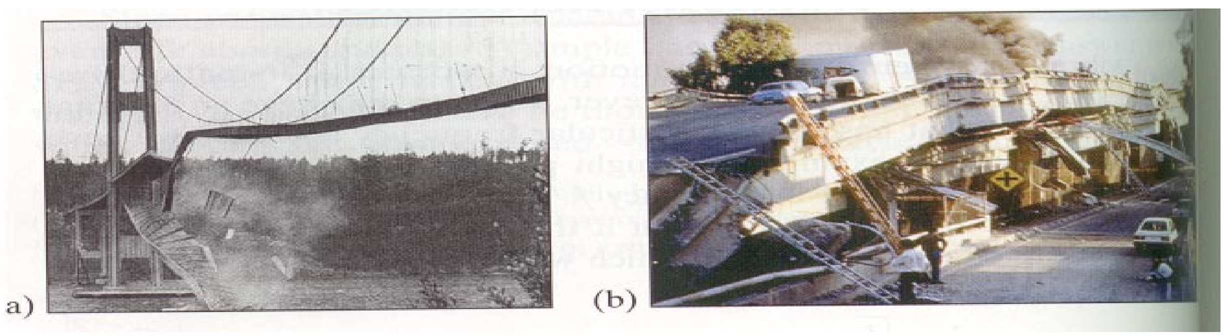


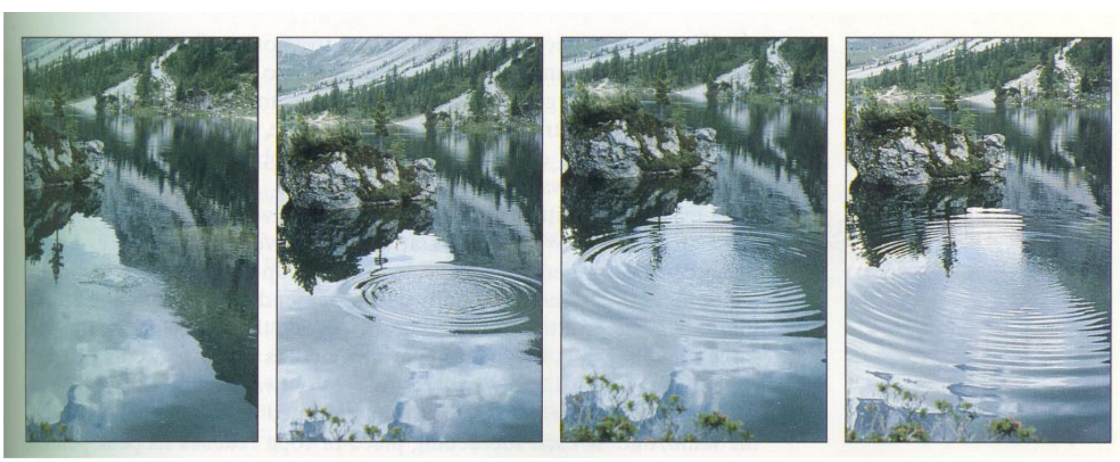
FIGURE 11–18 Resonance for lightly damped (A) and heavily damped (B) systems.

- The A can become VERY large when the driving f is near the natural frequency, f_o
    - A singer hits a note that is extremely close to the f_o of Pb atoms in combination with SiO_2 in lead crystal—what happens?

- Material objects are, in general, elastic—really important in building.
    - Railroad bridges have been reported to collapse due to a nick in a wheel set up a resonant vibration in the bridge.
    - Marching soldiers break step when crossing a bridge to avoid the possiblity of a similar catastrophe.
    - Tacoma Narrows Bridge collapsed in 1940 due to resonance of the bridge in tune with wind gusts (a)
    - The Oakland freeway collapse in 1989 due to seismic vibrations attaining its resonant frequency (b)



11.6 WAVE MOTION



Water waves [ripples] in response to a stone being tossed into a lake.

If you've ever seen leaves or floating objects on the water while waves are surging through the water, you've witnessed the fact that the leaf is not brought forward, it simply oscillates about it's equilibrium position. You may have thought that waves approaching the beach brought water into the beach, not so! The water molecules simply oscillate about their equilibrium positions.

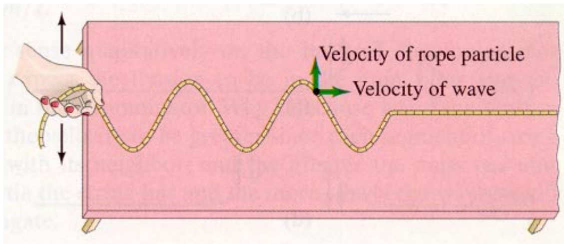
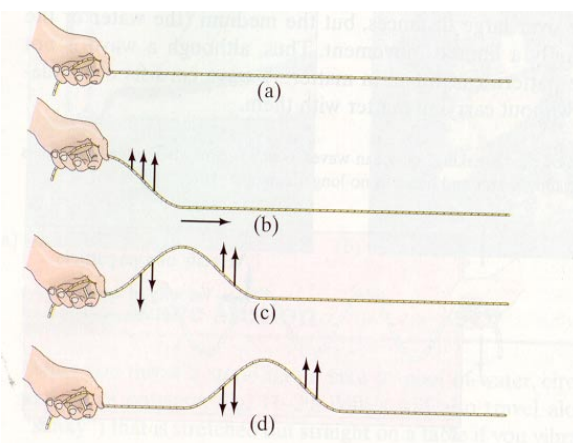


FIGURE 11–21 Wave traveling on a cord. The wave travels to the right along the cord. Particles of the cord oscillate back and forth on the tabletop.

Wave can move over large distances, but the medium [water, rope, etc] itself has limited movement. Although the wave is NOT matter, the wave pattern can travel in matter. *A wave consists of oscillations that move without carring matter with them.*

Example 11.9

Is the velocity of a wave moving along a cord the same as the velocity of a particle of the cord?



- Waves carry E from one place to another
- **pulse**—single wave bump—toss a rock into water, wind on ocean surface, quick jerk up and down on a rope
- **continuous or periodic wave**—has a source of disturbance that is continuous and oscillating; the source is a vibration or oscillation
- The source of any wave is a vibration—it is therefore a vibration that propagates outward and thus constitutes the wave
- If source is SHO the nthe wave [in a perfectly elastic media] is sinusoidal in both space and time
  - space—wave has shape of sine or cosine function
  - time—look @ motion of medium @ one place over a long time [between 2 posts at a pier perhaps]. The up and down motion of the water will be harmonic, up and down sinusoidally.

The Anatomy of a Wave

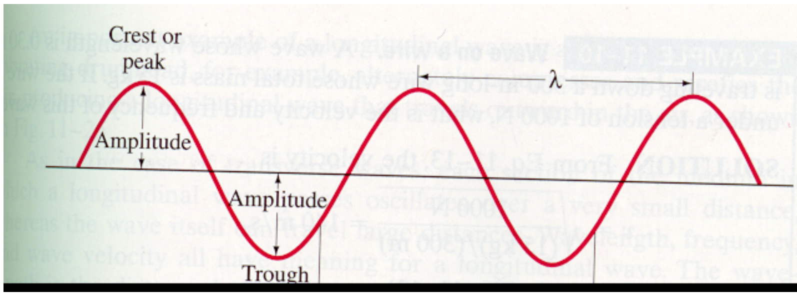


FIGURE 11–23

crest—peak of wave
trough—valley of wave
amplitude—point of maximum displacement [occurs at crest OR trough] from the equilibrium position
wavelength, λ--the distance from crest to crest OR trough to trough.

The speed of a wave: $v = \lambda f$

If the speed is constant (light, sound, etc.) then as λ increases, f must decrease and the converse is also true!

- v depends on the properties of the medium

$$v = \sqrt{\frac{F_T}{m/L}}$$

- v of a wave on a stretched string depends on F_T and on string's mass per unit length:

- F_T increases, v increases—very little space between each segment of the string
- m/L increases, inertia increases therefore the propagation slows

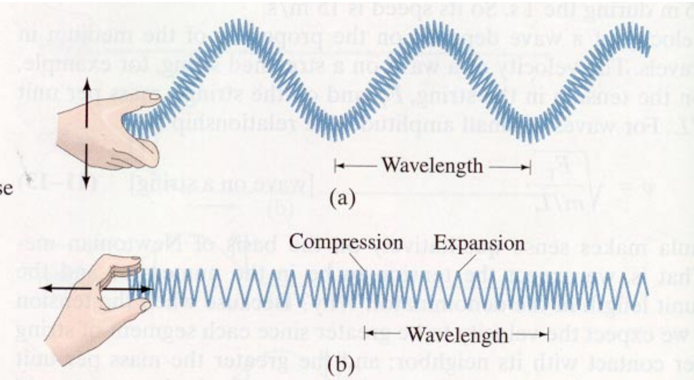
Example 11.10

A wave whose wavelength is 0.30 m is traveling down a 300 m long wire whose total mass is 15 kg. If the wire is under a tension of 1000 N, what is the velocity and frequency of this wave?

11.7TYPES OF WAVES: TRANSVERSE AND LONGITUDINAL

transverse—vibration of particles of the medium is perpendicular to the wave motion
longitudinal—vibration of particles of the medium is along the same motion of the wave
compressions—coils close
rarefactions (expansions)—coils far apart; separated

FIGURE 11–24 (a) Transverse wave; (b) longitudinal wave.



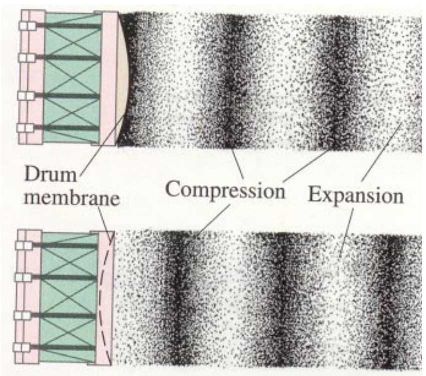


FIGURE 11–25 Production of a sound wave, which is longitudinal, shown at two moments in time, about a half period ($\frac{1}{2}T$) apart.

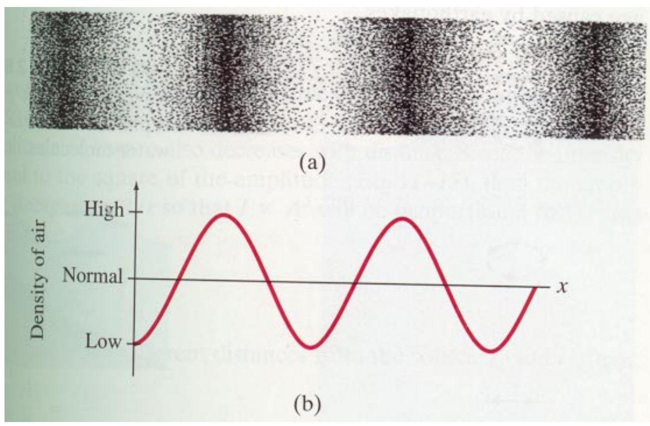


FIGURE 11–26 (a) A longitudinal wave with (b) its graphical representation, at a particular instant in time.

Longitudinal waves represented graphically by plotting the density of air molecules (or coils of slinky) vs. position at a given instant.

- Earthquakes
  - Both transverse and longitudinal waves involved
  - transverse—travel through the body of the Earth; called S waves for shear
  - longitudinal—P waves for pressure
- Both transverse and longitudinal travel through solid since atoms/molecules can vibrate about relatively fixed positions in any direction
- In a fluid, only longitudinal waves can propagate. Any transverse motion experiences no F_r since fluids flow
- Surface waves—travel along boundary between 2 materials and cause the most damage in an Earthquake