Fluid Mechanics Review

1. What is the density of water in kg/m^3? $1000\ kg/m^3$

2. What is the specific gravity of a material that has a density of $2700\ kg/m^3$? 2.7

3. A force of 100 N is applied over a square that measures 2 cm on a side. What is the pressure exerted on the square?

$$P = \frac{F}{A} = \frac{100\ N}{(.02m)^2} = 2.5 \times 10^5\ Pa$$

4. 200 Pa = 2×10^7 atm.

5. What is the gauge pressure at a depth of 20 meters under water? What is the absolute pressure?

$$P_g = \rho g h = (1000\ kg/m^3)(10\ m/s^2)(20m) = 200{,}000\ Pa$$

$$P_{ab} = P_g + P_{at.} = 200{,}000\ Pa + 1 \times 10^5\ Pa = 3.0 \times 10^5\ Pa$$

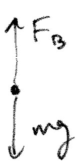
6. A hydraulic jack is used to lift a car that has a mass of 1200 kg. The area of the jack that the car rests on is has an area of 2.0 m^2. The area of the small piston of the jack is 0.04 m^2. What force must be applied to the small piston to lift the car?

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad F_1 = \frac{F_2 A_1}{A_2} = \frac{(12000N)(0.04m^2)}{2.0\ m^2} = 240\ N$$

7. A cube of aluminum measures 0.1 meter on a side. The specific gravity of aluminum is 2.7. The cube is dropped into a container of water.

   a. What is the buoyant force on the cube?

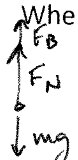
$$F_B = \rho_F V_o g = (1000\ kg/m^3)(0.1m)^3(10\ m/s^2) = 10N$$



   b. What is the acceleration of the cube when it enters the water? $m = \rho V = (2700\ kg/m^3)(.1m)^3 = 2.7\ kg$

$$a = \frac{\Sigma F}{m} = \frac{mg - F_B}{m} = \frac{(2.7\ kg)(10\ m/s^2) - 10N}{2.7\ kg} = 6.3\ m/s^2$$

   c. When the cube comes to rest on the bottom of the container, what is the normal force on the cube?



$$F_N = mg - F_B = (2.7\ kg)(10\ m/s^2) - 10\ N$$

$$F_N = 17\ N$$

8. The specific gravity of a material is 0.75. What percent of an object made from this material will float above the surface of water?

$$\text{Fraction Submerged} = \frac{\rho_o}{\rho_w} = \frac{750\ kg/m^3}{1000\ kg/m^3} = \frac{3}{4}$$

∴ 75% submerged.
∴ 25% above

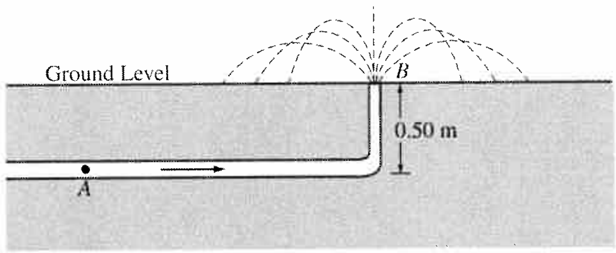
9. Water is flowing through a pipe with a cross-sectional area of 0.2 m^2 at a speed of 10 m/s. The pipe then narrows to a cross-sectional area of 0.1 m^2. What is the speed of the water through this section of the pipe?

$$A_1 v_1 = A_2 v_2 \qquad \therefore\ v_2 = \frac{A_1 v_1}{A_2} = \frac{(0.2\ m^2)(10\ m/s)}{0.1\ m^2} = 20\ m/s$$

10. What is Bernoulli's equation?

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

11.



An underground pipe carries water of density 1000 kg/m³ to a fountain at ground level, as shown above. At point *A*, 0.50 m below ground level, the pipe has a cross-sectional area of 1.0×10^{-4} m². At ground level, the pipe has a cross-sectional area of 0.50×10^{-4} m². The water leaves the pipe at point *B* at a speed of 8.2 m/s.

(a) Calculate the speed of the water in the pipe at point *A*.

$$A_A V_A = A_B V_B \quad \therefore \quad V_A = \frac{A_B V_B}{A_A} = \frac{(0.5\times10^{-4}\,m^2)(8.2\,m/s)}{1\times10^{-4}\,m^2} = \boxed{4.1\,m/s = V_A}$$

(b) Calculate the absolute water pressure in the pipe at point *A*.

$$P_A + \tfrac{1}{2}\rho V_A^2 + \rho g h_A = P_B + \tfrac{1}{2}\rho V_B^2 + \rho g h_B$$

$$P_A = P_B + \tfrac{1}{2}\rho(V_B^2 - V_A^2) + \rho g(h_B - h_A)$$

$$P_A = 1\times10^{-5}\,Pa + \tfrac{1}{2}(1000\,kg/m^3)(8.2^2 - 4.1^2) + (1000)(10)(0.5 - 0\,m)$$

$$\boxed{P_A = 1.3\times10^5\,N/m^2}$$

(c) Calculate the maximum height above the ground that the water reaches upon leaving the pipe vertically at ground level, assuming air resistance is negligible.

$$mgh = \tfrac{1}{2}mv_B^2 \qquad h = \frac{V_B^2}{2g} = \frac{(8.2\,m/s)^2}{(2)(10\,m/s^2)} \qquad \boxed{h = 3.4\,m}$$

(d) Calculate the horizontal distance from the pipe that is reached by water exiting the pipe at 60° from the level ground, assuming air resistance is negligible.

8.2 m/s, 60°

$$V_x = 8.2\,m/s\cos 60° = 4.1\,m/s$$

$$V_y = 8.2\,m/s\sin 60° = 7.1\,m/s$$

$$\text{time in air} = \frac{\Delta V}{a} = \frac{-7.1\,m/s - 7.1\,m/s}{-10\,m/s^2} = 1.42\,s$$

$$\Delta x = v_x t = (4.1\,m/s)(1.42\,s) = \boxed{5.8\,m = \Delta x}$$

12. A large water tank has a small hole punched into it 10 meters below the surface of the water. With what speed does the water flow out of the hole?

$$V = \sqrt{2gh} = \sqrt{(2)(10\,m/s^2)(10\,m)} = \boxed{14.1\,m/s = V}$$