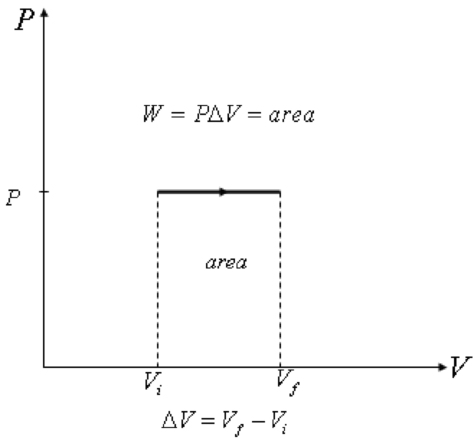
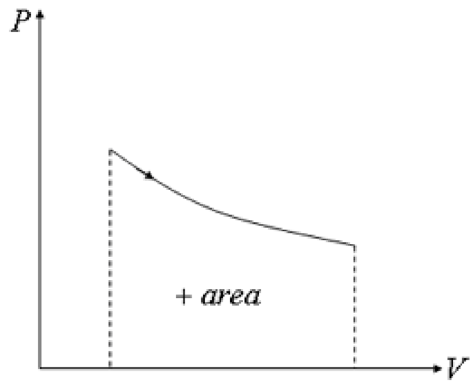
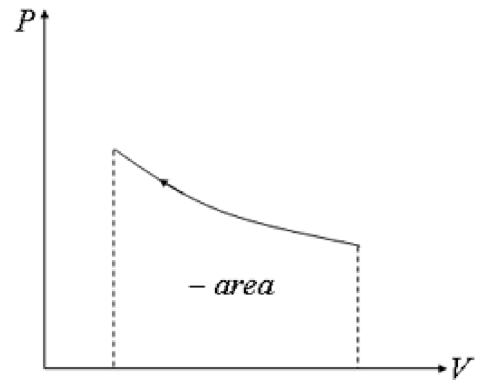
An Introduction to PV Diagrams

Consider the graphs below, they are quite popular on the AP Physics B Exam!



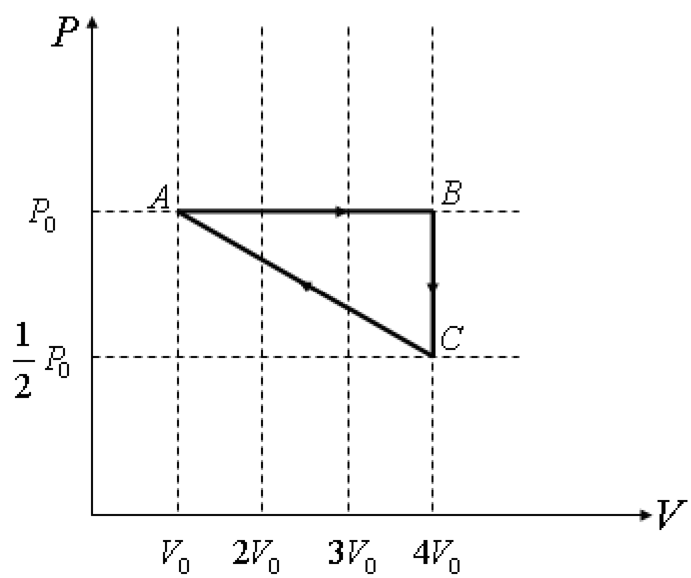




Work = W = area under the curve = $-P\Delta V$, note the direction of the isobar…the area is (+) for work done *by* the gas (expansion) and (–) for work done *on* the gas (compression).

A special case is that of a **cycle** in which the system is brought back to its original state after going through several processes. These are referred to as *P-V* diagrams and actually quite easy once you practice a few!

Example: One mole of monatomic ideal gas is enclosed under a frictionless piston. A series of processes occur, and eventually the state of the gas returns to its initial state with a *P-V* diagram as shown below. Answer the following in terms of P_0, V_0, and R. (That means no numbers, just variables!)



(a) Find the temperature at each vertex.
(b) Find the change in internal energy for each process.
(c) Find the work by the gas done for each process.

ANSWER:

(a) Find the temperature at each vertex .

"One mole" means $n = 1$ in the Ideal Gas Law, so substitute the values celebrating that $n = 1$, thus it "drops out" of the expression.

$$PV = nRT \;\; and \;\; n = 1$$

$$\therefore P_oV_o = RT_A \therefore T_A = \frac{P_oV_o}{R}$$

Similarly,

$$P_oV_o = RT_B \therefore T_B = \frac{P_o 4V_o}{R} = \frac{4P_oV_o}{R} \;\; and$$

$$P_oV_o = RT_C \therefore T_C = \frac{P_o 2V_o}{R} = \frac{2P_oV_o}{R}$$

(b) Find the change in internal energy for each process.

Since the internal energy for the monatomic ideal gas depends only on temperature, the *change* in internal energy for each process depends only on the temperature difference that occurs during the process:

$$\Delta U = \frac{3}{2}RT$$

$$\therefore \Delta U_{A\to B} = \left(\frac{3}{2}R\right)\Delta T = \left(\frac{3}{2}R\right)T_B - T_A = \left(\frac{3}{2}\cancel{R}\right)\left(\frac{4P_oV_o}{\cancel{R}} - \frac{P_oV_o}{\cancel{R}}\right) = \frac{3}{2}(3P_oV_o) = \frac{9}{2}P_oV_o$$

Similarly,

From the answers to Part (a), the change in temperature from B to C is $-2P_oV_o$, so

$$\Delta U = \frac{3}{2}RT$$

$$\therefore \Delta U_{B\to C} = \left(\frac{3}{2}R\right)\Delta T = \left(\frac{3}{\cancel{2}}\cancel{R}\right)\left(-\frac{\cancel{2}P_oV_o}{\cancel{R}}\right) = -3P_oV_o$$

The change in temperature from C back to A is $-P_oV_o$, so

$$\Delta U = \frac{3}{2}RT$$

$$\therefore \Delta U_{C\to A} = \left(\frac{3}{2}R\right)\Delta T = \left(\frac{3}{2}\cancel{R}\right)\left(-\frac{P_oV_o}{\cancel{R}}\right) = -\frac{3}{2}P_oV_o$$

(c) Find the work by the gas done for each process.

To find the work done by the gas, find the area under each segment, remembering the sign convention.

$W_{A\to B}$ = *area* under that segment = $3P_oV_o$

$W_{B\to C}$= *area* under that segment = 0

$W_{C\to A}$ = *area* under that segment = −(area of rectangle + area of triangle)

Why a negative sign? The gas is compressed!

$$= \frac{1}{2}(3V_o)\left(\frac{1}{2}P_o\right) + \left(\frac{1}{2}P_o\right)(3V_o) = -\frac{9}{4}P_oV_o$$

There are two things to note about part (c) that are true in general:

1. For a constant volume process like B → C, no work is done by the gas.
2. The total work done for the entire cycle is the area enclosed within the graph. In this example, the sum of the work is $W_{total} = 3P_oV_o - \frac{9}{4}P_oV_o = \frac{3}{4}P_oV_o$, the same as the area of the enclosed triangle.