1996 **Question 7 Scoring Guide** (10 points)

Solutions	**Distribution of points**
(a) 4 points	
For using the ideal gas equation, $pV = nRT$	1 point
From the fact that V is constant, it can be determined that p/T is a constant.	
$p_i/T_i = p_f/T_f$	1 point
Solving for the final pressure	
$p_f = T_f\, p_i/T_i$	
For the correct substitutions	1 point
$p_f = (298\ \text{K})(1\ \text{atm})/(363\ \text{K})$	
For the correct answer	1 point
$p_f = 0.82\ \text{atm} = 8.2 \times 10^4\ \text{Pa}$	
(b) 3 points	
For using the definition of pressure	1 point
$p = F/A$	
Solving for the force	
$F = pA$	
For the correct substitutions	1 point
$F = (8.2 \times 10^4\ \text{Pa})(0.005\ \text{m}^2)$	
For the correct answer	1 point
$F = 410\ \text{N}$	
(c) 3 points	
The number of moles of gas in a sample can be determined using the ideal gas equation.	
$pV = nRT$	
Solving for the number of moles	
$n = \dfrac{pV}{RT}$	
$\Delta n = n_f - n_i$	1 point
For realizing that volume and temperature are constant	1 point
$\Delta n = \dfrac{(\Delta p)\,V}{RT}$	
$\Delta n = \dfrac{(1\ \text{atm} - 0.82\ \text{atm})(0.005\ \text{m}^2)(0.15\ \text{m}^2)}{(8.31\ \text{J/mol}\cdot\text{K})(298\ \text{K})}$	
For the correct answer	1 point
$\Delta n = 5.45 \times 10^{-3}\ \text{mol}$	

Question 6

10 points total

6. (a) **3 points** — **Distribution of Points**

For using the definition of pressure — **1 point**

$P = \frac{F}{A}$

Calculating the additional pressure due to the weight of the block:

$\Delta P_2 = \frac{(2.50\ \text{kg})(9.8\ \text{m/s}^2)}{1.20\times10^{-2}\ \text{m}^2} = 2.04\times10^3\ \text{Pa}$ (or 2.08×10^3 Pa using $g = 10\,\text{m/s}^2$)

For adding the additional pressure to the pressure in state 1 — **1 point**

$P_2 = P_1 + \Delta P_2$

$P_2 = 1.02\times10^5\ \text{Pa} + 2.04\times10^3\ \text{Pa}$

For the correct answer — **1 point**

$P_2 = 1.04\times10^5\ \text{Pa}$

Alternate Solution — *Alternate points*

For determining the force on the piston in state 1 — *1 point*

$F_1 = P_1 A$

$F_1 = (1.02\times10^5\ \text{Pa})(1.20\times10^{-2}\ \text{m}^2) = 1224\ \text{N}$

Calculating the force due to the block:

$\Delta F_2 = (2.50\ \text{kg})(9.8\,\text{m/s}^2) = 24.5\ \text{N}$

For adding these two forces — *1 point*

$F_2 = F_1 + \Delta F_2$

$F_2 = 1224\ \text{N} + 24.5\ \text{N} = 1250\ \text{N}$

Calculating the pressure in state 2:

$P_2 = \frac{F_2}{A} = \frac{1250\ \text{N}}{1.20\times10^{-2}\ \text{m}^2}$

For the correct answer — *1 point*

$P_2 = 1.04\times10^5\ \text{Pa}$

Question 6 (cont.)

6. (b) **1 point** — **Distribution of Points**

At constant temperature, pressure times volume is a constant:

$P_1V_1 = P_2V_2$

Solving for V_2:

$$V_2 = \frac{P_1V_1}{P_2}$$

For correct substitutions — **1 point**

$$V_2 = \frac{\left(1.02\times10^5\ \text{Pa}\right)\left(1.50\times10^{-3}\ \text{m}^3\right)}{1.04\times10^5\ \text{Pa}}$$

$V_2 = 1.47\times10^{-3}\ \text{m}^3$

Alternate Solution — *Alternate points*

Applying the ideal gas law to state 1:

$P_1V_1 = nRT_1$

Solving for n:

$$n = \frac{P_1V_1}{RT_1}$$

$$n = \frac{\left(1.02\times10^5\ \text{Pa}\right)\left(1.50\times10^{-3}\ \text{m}^3\right)}{\left(8.31\ \text{J/mol}\bullet\text{K}\right)\left(273\ \text{K}\right)} = 6.74\times10^{-2}\ \text{mol}$$

Applying the ideal gas law to state 2:

$P_2V_2 = nRT_2$

Solving for V_2:

$$V_2 = \frac{nRT_2}{P_2}$$

For correct substitutions — *1 point*

$$V_2 = \frac{\left(6.74\times10^{-2}\ \text{mol}\right)\left(8.31\ \text{J/mol}\bullet\text{K}\right)\left(273\ \text{K}\right)}{1.04\times10^5\ \text{Pa}}$$

$V_2 = 1.47\times10^{-3}\ \text{m}^3$

Question 6 (cont.)

6. (c) **2 points**	**Distribution of Points**
For indicating the process is isobaric	**1 point**
For any explanation indicating that the pressure does not change from state 1 to state 2	**1 point**

6. (d) **2 point**

For indicating "yes" (the process is isobaric)	**1 point**
For any explanation indicating that the pressure does not change from state 4 to state 1	**1 point**

6. (e) **2 points**

At constant pressure, volume divided by temperature is constant

$$\frac{V_1}{T_1} = \frac{V_4}{T_4}$$

Solving for V_4:

$$V_4 = \frac{V_1 T_4}{T_1}$$

For correctly converting the temperatures to Kelvin	**1 point**
For correct substitutions	**1 point**

$$V_4 = \frac{\left(1.50\times10^{-3}\ \text{m}^3\right)\left(373\ \text{K}\right)}{273\ \text{K}}$$

$$V_4 = 2.05\times10^{-3}\ \text{m}^3$$

Alternate Solution	*Alternate points*

Applying the ideal gas law to state 4:

$$P_4 V_4 = nRT_4$$

Solving for V_4:

$$V_4 = \frac{nRT_4}{P_4}$$

For correctly converting the temperature to Kelvin	*1 point*
For correct substitutions (using n from part (b) and $P_4 = P_1$)	*1 point*

$$V_4 = \frac{\left(6.74\times10^{-2}\ \text{mol}\right)\left(8.31\ \text{J/mol}\bullet\text{K}\right)\left(373\ \text{K}\right)}{1.02\times10^{5}\ \text{Pa}}$$

$$V_4 = 2.05\times10^{-3}\ \text{m}^3$$

Question 6

10 points total | **Distribution of points**

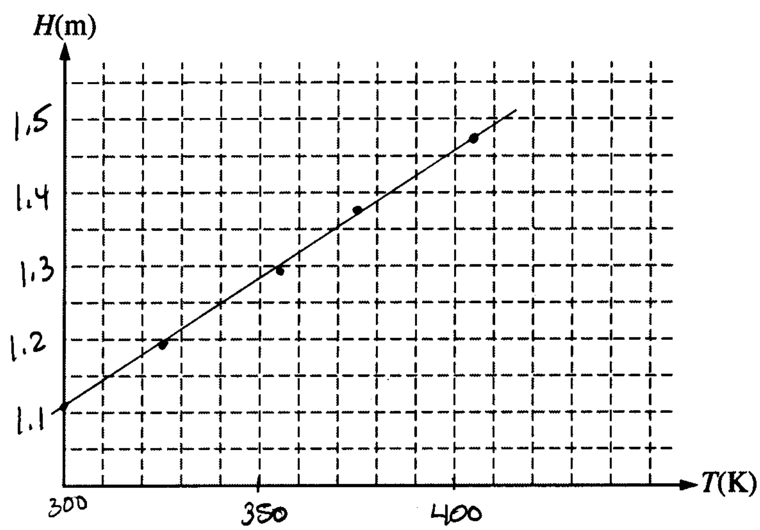
(a) 2 points

For writing the ideal gas equation of state — 1 point
$PV = nRT$
For an indication that $V = AH$ — 1 point
$PAH = nRT$
$H = \frac{nR}{PA}T$
Note: Simply writing $PAH = nRT$ or the equivalent earned both points. If the student tried to rearrange the equation, algebra mistakes were not penalized in this part.

(b) 4 points

For labeling both axes with linear numerical scales — 1 point
For having neither axis labeled with its scale starting at zero (no penalty for showing zero at the end of an axis and a "break" in the axis) — 1 point
For accurately plotting five data points that closely fit a straight line with a positive slope — 2 points
One point was lost if some points were inaccurately plotted.
Both points were lost if the data points were not visible, even if a line was drawn.

Example answer shown below. The question did not ask for a best-fit line, and it was not required for this part. However, a line is shown in the example, since it could be used in the determination of *n*.



Question 6 (continued)

Distribution of points

(c) 4 points

From part (a), $H = \frac{nR}{PA}T$

For any clear indication that the student used more than one data point — 1 point

For example: Setting up a slope calculation using subtraction, averaging calculated values for n or the ratio H/T, or using a linear regression on the calculator

For the correct slope of a best-fit line through the data points (this point not awarded if slope method not used) — 1 point

For correct substitutions into a correct expression containing n using consistent units for the values of P, R, A, and H — 1 point

Example using the line shown above, which happens to go through the first and last data points

$$\text{Slope of line} = \frac{nR}{PA}$$

$$\text{Slope} = \frac{(1.47 - 1.11)\,\text{m}}{(405 - 300)\,\text{K}} = \frac{0.36\ \text{m}}{105\ \text{K}}$$

$$n = \frac{PA}{R}(\text{slope})$$

$$n = \frac{\left(1 \times 10^5\ \text{Pa}\right)\left(0.027\ \text{m}^2\right)}{8.31\ \text{J}/(\text{mol}\bullet\text{K})}\left(\frac{0.36\ \text{m}}{105\ \text{K}}\right)$$

For a numerical answer that follows from substitutions into the correct expression above — 1 point

$n = 1.11$ moles

Question 5

10 points total | **Distribution of points**

(a)

(i) 3 points

For recognition that one component of the force is $P_{atm}A$ — 1 point

For recognition that another component of the force is Mg — 1 point

$F = P_{atm}A + Mg$

For $A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$ — 1 point

$$F = \frac{P_{atm}\pi D^2}{4} + Mg$$

(ii) 1 point

For $P = \frac{F}{A}$, with answer to (a) substituted for F — 1 point

$$P_{abs} = \frac{\frac{P_{atm}\pi D^2}{4} + Mg}{\frac{\pi D^2}{4}}$$

$$P_{abs} = P_{atm} + \frac{4Mg}{\pi D^2}$$

(b) 3 points

For checking the "Pressure goes up" answer space — 1 point

For a correct and complete justification, 2 points were awarded, with partial credit given where appropriate

Example:

For indicating that if heat is added, then the temperature must increase, recognizing that the volume is constant — 1 point

For then using the ideal gas law to show that with the volume constant, an increase in temperature implies an increase in pressure — 1 point

Alternate example — *Alternate points*

For indicating that if heat is added, then the internal energy and thus the kinetic energy of the gas molecules must increase, recognizing that the volume is constant — *1 point*

For then indicating that as the kinetic energy of the gas molecules increase, they exert more force on the walls of the cylinder, thus increasing the pressure — *1 point*

Question 5 (continued)

Distribution of points

(c) 3 points

For correct equation for work in terms of force and distance — 1 point

$W = Fx$

For correct substitution of x_0 for x — 1 point

$W = Fx_0$

For substitution of force from part (a)(i) — 1 point

$$W = \left(\frac{P_{atm}\pi D^2}{4} + Mg\right)x_0$$

Alternate solution — *Alternate points*

For correct equation for work in terms of pressure and volume change — *1 point*

$W = P_{abs}\,\Delta V$

For correct expression for ΔV — *1 point*

$$\Delta V = x_0 A = x_0\frac{\pi D^2}{4}$$

$$W = P_{abs}x_0\left(\frac{\pi D^2}{4}\right)$$

For substitution of absolute pressure from part (a)(ii) — *1 point*

$$W = \left(P_{atm} + \frac{4Mg}{\pi D^2}\right)x_0\left(\frac{\pi D^2}{4}\right)$$

$$W = \left(\frac{P_{atm}\pi D^2}{4} + Mg\right)x_0$$

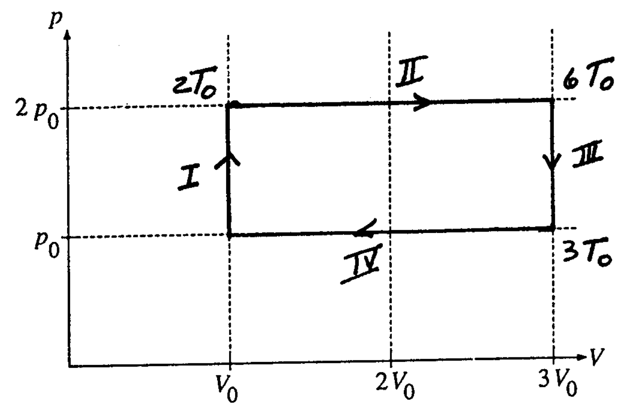
P-V DIAGRAMS

1983 Physics B Solutions

	Distribution of points
4. (a) 4 points	
An ideal gas obeys the equation $pV = nRT$.	
Since T is constant in process AB,	
$p_B V_B = p_0 V_0$	2 points
But $V_B = 2V_0$, so	
$p_B \cdot 2V_0 = p_0 V_0$	1 point
and $p_B = p_0/2$	1 point
(b) 3 points	
The first law of thermodynamics states that	
$\Delta U = Q - W$	1 point
where ΔU = change in internal energy; Q = heat absorbed by the system; W = work performed by the system	
Since the internal energy of an ideal gas depends only on its temperature, and process AB is isothermal,	
$\Delta U = 0$	1 point
Therefore $W = Q = 1000$ joules performed by the gas.	1 point
(c) 3 points	
The entropy increases during process AB.	1 point
This occurs because $\Delta S = \frac{Q}{T}$,	1 point
and Q, the heat absorbed, is positive.	1 point
Alternate Solution to (c):	*(Alternate points)*
Entropy is a measure of disorder.	(1 point)
It increases if the gas molecules have more kinetic energy or if they have a greater volume in which to move.	(1 point)
Since process AB is isothermal, there is no kinetic energy change, but the increase in volume leads to an increase in entropy.	(1 point)
(d) 3 points	
The second law of thermodynamics implies that for a reversible heat engine	
$\frac{Q_c}{T_c} = \frac{Q_h}{T_h}$	1 point
So $Q_c = \frac{T_c}{T_h} Q_h = \frac{200}{500} \cdot 1000 = 400$ J	2 points
(e) 2 points	
The work performed is positive.	1 point
There are many valid explanations, such as: (1) The area within the cycle $ABCDA$, traversed clockwise, represents work performed by the gas. (2) The work performed by the gas (area under ABC) is greater than the work performed on the gas (area under CDA) (3) In one cycle, $\Delta U = 0$ and $Q = Q_h - Q_c = 600$ J. By the first law of thermodynamics $W = Q - \Delta U = 600$ J, which is positive.	1 point

1989 Physics B

4.
(a)



i. 3 points

For at least one process diagrammed correctly	1 point
For all four processes diagrammed correctly	1 point
For cycle direction indicated correctly by arrows or Roman numeral labels	1 point

ii. 3 points

1 point awarded for each correctly labelled point <u>except</u> the initial point (T_0)	3 points

If none of the three points is correct, but evidence of the use of $PV \propto T$ was shown, 1 point was awarded.

1989 Physics B

Distribution of points

4. (continued)

(b)

i. 2 points

For indication of $W = p\Delta V$ or area under a curve — 1 point

$W = p_0(2V_0) = 2p_0V_0$ — 1 point

ii. 2 points

$\Delta E = 0$ for a cycle — 2 points

iii. 2 points

$\Delta E = Q - W$ or $Q = W$ — 1 point

$Q = 2p_0V_0$ — 1 point

In each of the three parts, if none of the indicated points were earned, 1 point was awarded for an indication of summing relevant quantities for the four processes, e.g., $W_I + W_{II} + W_{III} + W_{IV}$

(c) 3 points

For any indication that only C_p is applicable in direct calculation of Q — 1 point

For relation between Q and ΔT: $Q = nC_p\Delta T$ — 1 point

$Q = \frac{5}{2}nR\ 4T_0 = 10\ nRT_0$

$nRT_0 = p_0V_0$

$Q = 10p_0V_0$ — 1 point

Alternate solution — (Alternate points)

$\Delta E = Q - W$ — (1 point)

$nC_V\Delta T = Q - W$ — (1 point)

$\left(\frac{3}{2}nR\right)(4T_0) = Q - 4p_0V_0$

$Q = 6nRT_0 + 4p_0V_0$

$nRT_0 = p_0V_0$

$Q = 10p_0V_0$ — (1 point)

1990

4.

(a) 4 points

$pV = nRT$ 1 point

$T_A = \frac{(1 \times 10^5 \text{ N/m}^2)(25 \times 10^{-3} \text{ m}^3)}{(1 \text{ mole})(8.32 \text{ J/mole}\cdot\text{K})} = 300 \text{ K}$ 1 point

$T_B = \frac{(2 \times 10^5 \text{ N/m}^2)(25 \times 10^{-3} \text{ m}^3)}{(1 \text{ mole})(8.32 \text{ J/mole}\cdot\text{K})} = 600 \text{ K}$ <u>or</u> $T_B = 2T_A$ 1 point

$T_C = \frac{(1 \times 10^5 \text{ N/m}^2)(50 \times 10^{-3} \text{ m}^3)}{(1 \text{ mole})(8.32 \text{ J/mole}\cdot\text{K})} = 600 \text{ K}$ <u>or</u> $T_C = 2T_A$ 1 point

(b) 3 points

$W = p\Delta V$ <u>or</u> W = area under a curve 1 point

$W_{net} = \frac{1}{2}(1 \times 10^5 \text{ N/m}^2)(25 \times 10^{-3} \text{ m}^3)$ <u>or</u> W_{net} = area of triangle 1 point

$W_{net} = 1250 \text{ J}$ 1 point

<u>Alternate Solution</u> (Alternate Points)

$W_{AB} = 0$

$W_{BC} = \bar{p}\Delta V = (1.5 \times 10^5 \text{ N/m}^2)(25 \times 10^{-3} \text{ m}^3) = 3750 \text{ J}$

$W_{CA} = (1 \times 10^5 \text{ N/m}^2)(-25 \times 10^{-3} \text{ m}^3) = -2500 \text{ J}$

(+2500 J also acceptable)

For any one of the above values correct (1 point)

For a second of the above values correct (1 point)

$W_{net} = 3750 \text{ J} - 2500 \text{ J} = 1250 \text{ J}$ (1 point)

1990 Physics B

Distribution of points

(c) 3 points

$\Delta U = Q - W$ <u>or</u> $\Delta U = 0$	1 point
$Q_{net} = W_{net}$	1 point
$Q_{net} = 1250$ J	1 point
<u>Alternate Solution</u>	(Alternate Points)
$Q = nC\Delta T$	(1 point)
$Q_{AB} = nC_V(T_B - T_A) = 3740$ J	
$Q_{CA} = nC_p(T_A - T_B) = -6240$ J	
$\Delta U_{BC} = 0$,	(1 point)
so $Q_{BC} = W_{BC} = (1.5 \times 10^5 \text{ N/m}^2)(25 \times 10^{-3} \text{m}^3) = 3750$ J	
$Q_{net} = Q_{AB} + Q_{BC} + Q_{CA} = 3740 \text{ J} + 3750 \text{ J} - 6240 \text{ J} = 1250$ J	(1 point)

(d) 2 points

$Q_{CA} = nC_p\Delta T$ <u>or</u> $Q_{CA} = nC_V\Delta T + p\Delta V$	1 point
$Q_{CA} = (\pm)\ 6240$ J	1 point

(e) 2 points

efficiency $= \dfrac{W}{Q_{in}}$ <u>or</u> $\dfrac{Q_{in} \pm Q_{out}}{Q_{in}}$ <u>or</u> $\dfrac{W_{out}}{W_{in}}$	1 point
efficiency $= \dfrac{1250 \text{ J}}{7490 \text{ J}} = 0.17$	1 point

For correct units indicated in all answers — 1 point

1993 Physics B Solutions — **Distribution of Points**

5.

(a) 3 points

For any statement of the ideal gas law:

$pV = nRT$ — 1 point

Solving for T:

$T = \frac{pV}{nR}$

For substituting the correct values:

$T_b = \frac{(1.2 \times 10^5 \text{ Pa})(51 \times 10^{-3} \text{ m}^3)}{(1 \text{ mole})(8.32 \text{ J/mole}\cdot\text{K})}$ — 1 point

$T_b = 736$ K — 1 point

(Alternate Solution) — (Alternate Points)

For any expression of the law of Charles and Gay-Lussac:

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ — (1 point)

For substituting the correct values:

$T_2 = \frac{T_1 V_2}{V_1} = \frac{(250 \text{ K})(51 \times 10^{-3} \text{ m}^3)}{17 \times 10^{-3} \text{ m}^3}$ — (1 point)

$T_2 = 750$ K — (1 point)

(The difference in the answers from the two solutions results because values given in this problem were rounded to two significant figures.)

(b) 3 points

For any statement of the relationship between heat and change in temperature at constant pressure:

$Q = nC_p\Delta T$ — 1 point

For substituting the correct values:

$Q = (1 \text{ mole})\left(20.8 \frac{\text{J}}{\text{mole}\cdot\text{K}}\right)(736 \text{ K} - 250 \text{ K})$ — 1 point

$Q = 10{,}100$ J (or 10,400 J if $T_b = 750$ K) — 1 point

1993 Physics B Solutions **Distribution of Points**

5. (continued)
(c) 3 points

For any statement of the First Law of Thermodynamics:

$U_b - U_a = \Delta U = Q - W = Q - p\Delta V$ 1 point

For substituting correctly using the value of the heat from (b):

$\Delta U = 10{,}100 \text{ J} - (1.2 \times 10^5 \text{ Pa})(51 \times 10^{-3} \text{ m}^3 - 17 \times 10^{-3} \text{ m}^3)$ 1 point

$= 10{,}100 \text{ J} - 4{,}080 \text{ J}$

$\Delta U = 6{,}020 \text{ J}$ (or 6,320 J if $Q = 10{,}400$ J) 1 point

(Alternate Solution I) (Alternate Points)

For any statement of the relationship between the change in internal energy and temperature at constant volume:

$\Delta U = nC_v\Delta T$ (1 point)

For substituting the correct values:

$\Delta U = (1 \text{ mole})\left(12.5 \frac{\text{J}}{\text{mole}\cdot\text{K}}\right)(736 \text{ K} - 250 \text{ K})$ (1 point)

$\Delta U = 6{,}075 \text{ J}$ (or 6,250 J if $T_b = 750$ K) (1 point)

(Alternate Solution II) (Alternate Points)

For any statement of the change in internal energy of an ideal monatomic gas:

$\Delta U = \frac{3}{2} nR\Delta T$ (1 point)

For substituting the correct values:

$\Delta U = \frac{3}{2}(1 \text{ mole})(8.32 \text{ J/mole}\cdot\text{K})(736 \text{ K} - 250 \text{ K})$ (1 point)

$\Delta U = 6{,}065 \text{ J}$ (or 6,240 J if $T_b = 750$ K) (1 point)

(d) 1 point

The work done by the gas is equal to $p\Delta V$ (i.e. the area under the pV curve.)

In process bc, $\Delta V = 0$, Therefore:

$W = 0$ 1 point

(If 2 or more points were lost elsewhere in the problem, $W = p\Delta V$ was awarded 1 point here if it was not used in previous parts.)

1993 Physics B Solutions | **Distribution of Points**

5. (continued)

(e) 2 points

Using the First Law of Thermodynamics:

$\Delta U = Q - W$

The gas returns to its initial state, so $\Delta U = 0$. Therefore:

$W_{net} = Q_{net}$ — 1 point

$W = 1,800$ J — 1 point

(<u>Note</u>: 1,800 J is the correct value, to two significant figures, of the heat added to the gas that is consistent with the other information given in the problem. Students who correctly used the incorrect value of 3,300 J that was printed in the test booklet received full credit.)

(f) 3 points

For any statement of the Carnot efficiency of an engine:

$e = \frac{T_h - T_c}{T_h} = \frac{T_b - T_a}{T_b}$ — 1 point

For substituting the correct values:

$e = \frac{736\text{ K} - 250\text{ K}}{736\text{ K}}$ — 1 point

$e = 0.66$ or $\frac{2}{3}$ (or 0.67 if $T_b = 750$ K) — 1 point

1999 Physics B Solutions

Distribution of points

Question 7 (10 points)

(a) 2 points

For relating the thermodynamic states at point A and point C — 1 point

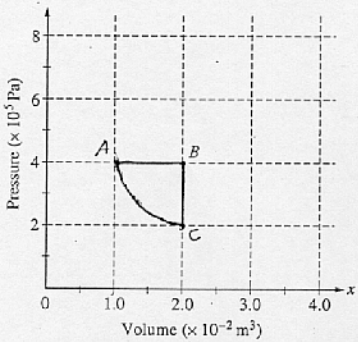
Examples: $\frac{p_A V_A}{T_A} = \frac{p_C V_C}{T_C}$ OR $p_A V_A = p_C V_C$

For finding the pressure at point C — 1 point

$$p_C = p_A \frac{V_A}{V_C} = (4 \times 10^5 \text{ Pa})\left(\frac{1.0 \times 10^{-2} \text{ m}^3}{2.0 \times 10^{-2} \text{ m}^3}\right)$$

$p_C = 2.0 \times 10^5$ Pa OR $p_C = 2.0$ atm

(b) 4 points



For correctly plotting and labeling Point A — 1 point

For correctly plotting and labeling Point C — 1 point

For indicating that path CA is along an isotherm — 1 point

For drawing a complete cycle with path AB shown as an isobaric process and path BC shown as an isochoric process. — 1 point

Question 7 (continued)

(c) 2 points

For statement that net work done by the gas is positive	1 point
For correct justification	1 point

Examples:

1. Work is related to area and the net area is positive.
2. More heat is absorbed in the cycle that is exhausted.
3. Expansion occurs at a higher pressure while compression occurs at a lower pressure.
4. Thermodynamic cycle is clockwise.

(d) 2 points

For statement that the device is a heat engine	1 point
For correct justification	1 point

Examples:

1. Work done by the system is positive.
2. Work done by the system is done at a higher temperature.
3. Heat is absorbed at a higher temperature and exhausted at a lower temperature.
4. Thermodynamic cycle is clockwise.

Question 5

10 points total | **Distribution of points**

(a) 1 point

For a correct calculation of the change in internal energy — 1 point

$U_a - U_c = \Delta U_{c\to a} = Q_{c\to a} + W_{c\to a}$

$\Delta U_{c\to a} = 685\text{ J} - 120\text{ J}$

$\Delta U_{c\to a} = 565\text{ J}$

(b) 3 points

i. (1 point)

For correct choice of heat removed from the gas — 1 point

ii. (2 points)

For recognition that the change in internal energy is opposite in sign from part (a) answer — 1 point

$\Delta U_{a\to b\to c} = -\Delta U_{c\to a} = -565\text{ J}$

Calculating the heat:

$Q_{c\to d\to a} = \Delta U_{c\to d\to a} - W_{c\to d\to a}$

$Q_{c\to d\to a} = -565\text{ J} - 75\text{ J}$

For the correct answer — 1 point

$Q_{c\to d\to a} = -640\text{ J}$

(c) 3 points

The total work done is the sum of the work for the two sections of the path

$W_{c\to d\to a} = W_{c\to d} + W_{d\to a}$

For some indication that the work done along path $c \to d$ is zero — 1 point

The work done along path $d \to a$ is the area under the curve

$W_{c\to d\to a} = 0 - P\,\Delta V$

For correct substitution — 1 point

$W_{c\to d\to a} = -(6.0\times 10^5\text{ Pa})(1.0\times 10^{-3}\text{ m}^3 - 0.75\times 10^{-3}\text{ m}^3)$

For the correct answer — 1 point

$W_{c\to d\to a} = -150\text{ J}$

(d) 3 points

For correct choice of heat added to the gas — 1 point

For a complete explanation that references the first law — 2 points

Example: Since $\Delta U_{c\to d\to a}$ is positive (i.e. 565 J) and work is done (i.e. -150 J), Q must be positive.

An incomplete argument with correct relevant assertions and no mistakes earned 1 point.
An incomplete argument with irrelevant or incorrect assertions earned no points.

Question 5

10 points total	**Distribution of points**
(a) 2 points	
For a statement of the ideal gas law	1 point
$pV = nRT$	
$(200\ \text{N/m}^2)(20\ \text{m}^3) = (1\ \text{mol})(8.32\ \text{J}/(\text{mol}\cdot\text{K}))T$	
For the correct answer	1 point
$T = 481\ \text{K}$	
(b) 2 points	
For indicating that the work W done on the gas is equal to the area enclosed by the cycle or for $W = -p\Delta V$	1 point
$W = $ area of triangle enclosed by cycle $= \frac{1}{2}bh = \frac{1}{2}(60\ \text{m}^3 - 20\ \text{m}^3)(400\ \text{N/m}^2 - 200\ \text{N/m}^2)$	
For the correct answer	1 point
$W = 4000\ \text{J}$	
(c) 2 points	
i. (1 point)	
For indicating that heat is removed from the gas during one complete cycle	1 point
ii. (1 point)	
Using the first law of thermodynamics	
$\Delta U = Q + W$	
Recognizing that $\Delta U = 0$ for a closed cycle	
$Q = -W$	
For the correct answer consistent with part (b)	1 point
$Q = -4000\ \text{J}$	
Note: Since the question could be interpreted as asking for the magnitude of the heat added to or remove from, the minus sign was not necessary for full credit.	

Question 5 (continued)

	Distribution of points
(d) 2 points	
For indicating that the internal energy of the gas after one cycle is the same as before	1 point
For a reasonable justification	1 point
Example: The internal energy of the gas is a function of the temperature and the temperature is the same at the beginning and end of each cycle.	
(e) 2 points	
For indicating that the entropy of the gas after one cycle is the same as before	1 point
For a reasonable justification	1 point
Example: The entropy is a function of the state of the gas, and after one complete cycle the gas has returned to its original state.	

Question 5

10 points total | **Distribution of points**

(a)

i. 2 points

For a correct calculation of the work done on the gas — 1 point

$W_{on} = -P\Delta V$

$W_{on} = -\left(600\ \text{N/m}^2\right)\left(9.0\ \text{m}^3 - 3.0\ \text{m}^3\right)$

$W_{on} = -3600\ \text{J}$

For recognition that the work done by the gas is the negative of the work done on the gas — 1 point

$W_{by} = 3600\ \text{J}$

ii. 3 points

For a correct expression or derivation of the expression for ΔU — 1 point

$\Delta U = \frac{3}{2}nR\ \Delta T$

For correct calculation of T's or ΔT using the ideal gas law, $PV = nRT$ — 1 point

$\Delta U = \frac{3}{2}(2\ \text{moles})\left(8.31\frac{\text{J}}{\text{mol K}}\right)(325\ \text{K} - 108\ \text{K})$

OR since $P\ \Delta V = nR\ \Delta T$, $\Delta U = \frac{3}{2}P\ \Delta V = \frac{3}{2}\left(600\ \text{N/m}^2\right)\left(9\ \text{m}^3 - 3\ \text{m}^3\right)$

For the correct answer — 1 point

$\Delta U = 5400\ \text{J}$

Note: The equation $\Delta U = \frac{3}{2}nR\ \Delta T$ can be derived from the expressions for K_{avg} and υ_{rms} found in the equation sheet as follows:

$U = NK_{avg}$, where N = number of molecules in the gas = nN_0

$U = nN_0\frac{3}{2}k_BT$

$\upsilon_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_BT}{\mu}}$, so $\frac{R}{M} = \frac{k_b}{\mu}$

$R = k_B\frac{M}{\mu} = k_BN_0$

$U = \frac{3}{2}nRT$

$\Delta U = \frac{3}{2}nR\ \Delta T$

Question 5 (continued)

Distribution of points

(a) continued

iii. 1 point

For correct substitution of answers from parts i. and ii. into the first law of thermodynamics — 1 point

$\Delta U = Q + W_{on}$

$Q = \Delta U - W_{on}$

$Q = 5400 \text{ J} - (-3600 \text{ J})$

$Q = 9000 \text{ J}$

Alternate Solutions for parts ii. and iii. — *Alternate points*

Solving part iii. first:

$$Q = nc_P\,\Delta T = n\frac{5}{2}R\,\Delta T = (2 \text{ moles})\frac{5}{2}\left(8.31\frac{\text{J}}{\text{mol K}}\right)(325 \text{ K} - 108 \text{ K}) = 9000 \text{ J}$$

For a correct equation — *1 point*

For correct calculation of T's or ΔT — *1 point*

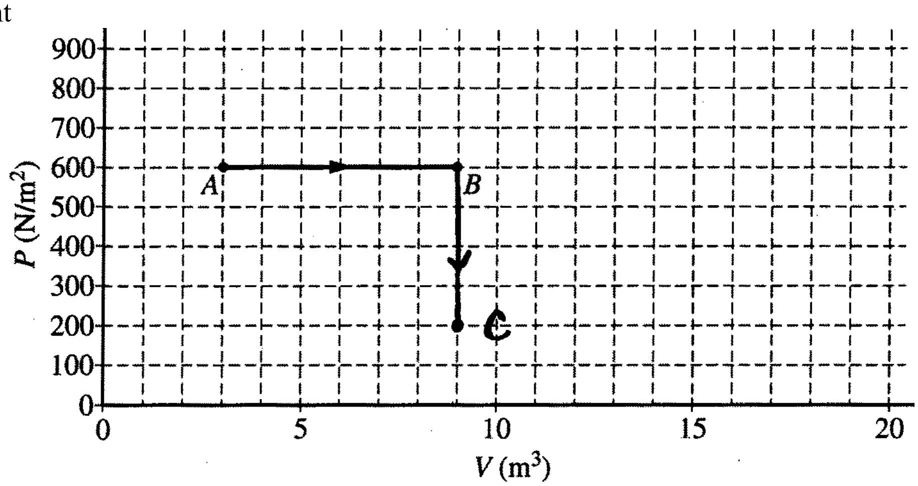
For the correct answer — *1 point*

Returning to solve part ii.:

$\Delta U = Q + W_{on} = 9000 \text{ J} + (-3600 \text{ J}) = 5400 \text{ J}$

For correct substitutions into the first law of thermodynamics of answers from parts i. and iii. — *1 point*

(b) 1 point

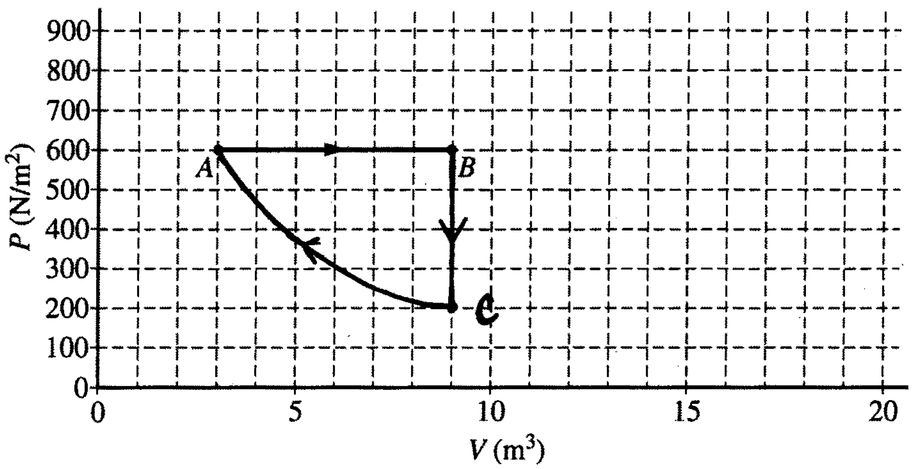


For point C plotted and labeled correctly as above, and for a correct straight line from point B to point C — 1 point

Distribution of points

(c)

i. 1 point



For a correct curve from point *C* to point *A*. Curve must be concave upward. 1 point

ii. 2 points

For correctly indicating that heat is removed from the gas 1 point

For a correct justification such as explaining in words or symbols that the change in internal energy is zero, so from first law of thermodynamics $Q = -W$. Since the work done on the gas is greater than zero, Q is negative. Therefore heat is removed from the gas. 1 point

Question 5

10 points total | **Distribution of points**

(a) 2 points

For recognizing that $P_A = P_B$ — 1 point

At point A: $P_1V_1 = nRT_1$

At point B: $P_1\dfrac{V_1}{2} = nRT_2$

Dividing these equations and solving for T_2:

$$\frac{P_1V_1}{P_1\dfrac{V_1}{2}} = \frac{T_1}{T_2}$$

$$2 = \frac{T_1}{T_2}$$

For the correct answer — 1 point

$T_2 = T_1/2$

(b) 2 points

Answer can be obtained by either comparing points B & C or points A & C using the ideal gas law at each point

For recognizing that $T_C = 2T_B$ OR $T_C = T_A$ — 1point

<u>Comparing points B & C</u>

At point B: $P_1\dfrac{V_1}{2} = nR\dfrac{T_1}{2}$

At point C: $P_2\dfrac{V_1}{2} = nRT_1$

Dividing the equations and solving for P_2:

$$\frac{P_1\dfrac{V_1}{2}}{P_2\dfrac{V_1}{2}} = \frac{nR\dfrac{T_1}{2}}{nRT_1}$$

$$\frac{P_1}{P_2} = \frac{1}{2}$$

For the correct answer

$P_2 = 2P_1$

<u>Comparing points C & A</u>

At point A: $P_1V_1 = nRT_1$

At point C: $P_2\dfrac{V_1}{2} = nRT_1$

Dividing the equations and solving for P_2:

$$\frac{P_1V_1}{P_2\dfrac{V_1}{2}} = 1$$

$$\frac{2P_1}{P_2} = 1$$

For the correct answer — 1 point

$P_2 = 2P_1$

Question 5 (continued)

Distribution of points

(c) 2 points

For correct equation for work done on the gas (regardless of sign) OR for recognition that work is the area under the line *AB* on the graph. (No work is done from *B* to *C* because the change in volume is zero.) — 1 point

$W = -P\,\Delta V$

$W = -P_1\left(\frac{V_1}{2} - V_1\right)$

For the correct answer including correct sign — 1 point

$W = \frac{P_1 V_1}{2}$

(d) 4 points

Heat was added to the gas in processes *BC* and *CA*, but not in *AB*.

For two or three correct indications of whether or not heat was added to the gas in each process (The absence of a check mark was taken as in indication that heat was not added.) — 1 point

For a reasonable justification for process *AB.* This point was awarded only if this process was indicated correctly (i.e., not checked) — 1 point

Example: The volume decreases so the work done on the gas is positive. The temperature decreases so the change in internal energy is negative. Therefore $Q = \Delta U - W$ is negative. Heat is expelled from the gas. (Note: Answer must mention work for credit.)

For a reasonable justification for process *BC.* This point was awarded only if this process was indicated correctly (i.e., checked) — 1 point

Example: There is no change in volume so no work is done. The temperature increases so the internal energy increases. Therefore $Q = \Delta U - W$ is positive. (This point also awarded for only referring to increasing temperature or for referring to increasing speed of the molecules.)

For a reasonable justification for process *CA*. This point was awarded only if this process was indicated correctly (i.e., checked). — 1 point

Example: There is no change in temperature or internal energy. The volume increases so work is done by the gas. Heat needs to be added to the gas to do this work.

Question 5

10 points total | **Distribution of points**

(a)

(i) 2 points

From the ideal gas law, $PV/T =$ a constant

Points A and B are on the isothermal, so they are at the same temperature.

Therefore, $P_B V_B = P_A V_A$

$P_B 2V_0 = P_A V_0$

For the correct answer — 1 point

$$\frac{P_B}{P_A} = \frac{1}{2}$$

For a correct justification (such as the reasoning shown above) — 1 point

(ii) 2 points

Points C and B are at the same pressure.

Therefore, $\dfrac{P_C}{P_A} = \dfrac{P_B}{P_A}$

For the correct answer — 1 point

$$\frac{P_C}{P_A} = \frac{1}{2}$$

For a correct justification (such as the reasoning shown above) — 1 point

(iii) 2 points

Points A and B are on the isothermal, so they are at the same temperature.

For the correct answer — 1 point

$$\frac{T_B}{T_A} = 1$$

For a correct justification (such as the reasoning shown above) — 1 point

Question 5 (continued)

	Distribution of points
(a) (continued)	
(iv) 2 points	
Points C and A are at the same volume.	
Therefore, from the ideal gas law $\frac{P_C}{P_A} = \frac{T_C}{T_A}$.	
$\frac{P_C}{P_A} = \frac{1}{2}$, which was determined in part (ii) above	
For the correct answer	1 point
$\frac{T_C}{T_A} = \frac{1}{2}$	
For a correct justification (such as the reasoning shown above)	1 point
(b) 1 point	
For a correct explanation	1 point
Internal energy depends only on the temperature. Since step I is isothermal there is no change in temperature and thus no change in internal energy	
(c) 1 point	
For a correct explanation	1 point
$W = -P\Delta V$. In step III there is no change in volume, and thus no work done.	

Question 5

10 points total | **Distribution of points**

(a) 3 points

Process	W	Q	ΔU
$A \rightarrow B$	0	+	+
$B \rightarrow C$	–	+	0
$C \rightarrow A$	+	–	–

For correctly identifying the signs of all three variables $(W,\ \Delta U,\ Q)$ for process $A \rightarrow B$ — 1 point

For correctly identifying the signs of all three variables $(W,\ \Delta U,\ Q)$ for process $B \rightarrow C$ — 1 point

For correctly identifying the signs of all three variables $(W,\ \Delta U,\ Q)$ for process $C \rightarrow A$ — 1 point

(b) 4 points

For a correct explanation of why zero work is done on the gas — 1 point

Examples of correct responses include:

- There is no area under the graph/curve.
- There is no volume change.
- The piston does not move.
- $W = -P\Delta V$, where $\Delta V = 0$.
- $W = 0$ by definition in an isochoric/isovolumetric process.

For a correct explanation of why ΔU is positive — 1 point

Examples of correct responses include:

- For a fixed number of moles of a sample, an increase in pressure at constant volume implies an increase in temperature, and an increased temperature implies an increase in internal energy (ΔU is positive).
- State *B* is on a higher isotherm than state *A*.
- $U = \frac{3}{2}nRT$; since T increases, ΔU is positive.
- $U = \frac{3}{2}PV$; since P increases at a constant V, ΔU is positive.
- A correct use of the first law of thermodynamics leading to ΔU being positive ($\Delta U = Q + W$, but $W = 0$ so $\Delta U = Q$; since Q is positive, ΔU must be also).

For a correct explanation of why Q is positive — 1 point

Examples of correct responses include:

- If pressure increases *and* volume is constant, heat must be added to system.
- If temperature increases *and* volume is constant, heat must be added to system.
- A correct use of the first law of thermodynamics leading to Q being positive ($\Delta U = Q + W$, but $W = 0$ so $\Delta U = Q$; since ΔU is positive, Q must be also).

For stating the first law of thermodynamics, whether used correctly or not, OR for correctly explaining all three variables without reference to the first law of thermodynamics — 1 point

Question 5 (continued)

Distribution of points

(c) 3 points

For correctly relating states *B* and *C* at a constant temperature, using either the ideal gas law or Boyle's Law — 1 point

$$\frac{P_B V_B}{T_B} = \frac{P_C V_C}{T_C} \quad \text{or} \quad P_B V_B = P_C V_C$$

For the correct substitution of the appropriate values — 1 point

$$V_C = \frac{P_B}{P_C} V_B = \left(\frac{5 \text{ atm}}{1 \text{ atm}}\right)\left(0.001 \text{ m}^3\right)$$

For the correct answer including units — 1 point

$$V_C = 0.005 \text{ m}^3$$

Alternate solution — *Alternate points*

For correctly relating states A and B at a constant volume, using the ideal gas law, to determine the temperature T_B — *1 point*

$$\frac{P_A V_A}{T_A} = \frac{P_B V_B}{T_B}$$

$$T_B = \frac{P_B}{P_A} T_A = \left(\frac{5 \text{ atm}}{1 \text{ atm}}\right)(400 \text{ K}) = 2000 \text{ K}$$

For realizing that $T_C = T_B$ *and correctly relating states A and C at constant pressure, using the value of* T_B *and the ideal gas law or Charles' Law to calculate* V_C. — *1 point*

$$\frac{P_A V_A}{T_A} = \frac{P_C V_C}{T_C} \quad \text{or} \quad \frac{V_A}{T_A} = \frac{V_C}{T_C}$$

$$V_C = \frac{T_C}{T_A} V_A = \frac{T_B}{T_A} V_A = \left(\frac{2000 \text{ K}}{400 \text{ K}}\right)\left(0.001 \text{ m}^3\right)$$

For the correct answer, including units — *1 point*

$$V_C = 0.005 \text{ m}^3$$

Another method is to use the ideal gas law to calculate T_B*, set* $T_C = T_B$*, and once again use the ideal gas law to calculate* V_C.

1986

Physics B

5.

(a) 2 points

$e = 1 - \frac{T_{cold}}{T_{hot}} \quad \left\{ \text{or } 1 - \frac{Q_{cold}}{Q_{hot}} \right\}$ — 1 point

$e = 1 - \frac{276\ K}{298\ K} = 0.074$

For correct temperature substitution — 1 point

(b) 5 points

$e = \frac{W_{useful}}{Q_{in}}$ — 1 point

$Q_{in} = \frac{W_{useful}}{e}$

$\frac{Q_{in}}{\Delta t} = \frac{1}{\Delta t}\left[\frac{W_{useful}}{e}\right] = \frac{100\ MW}{(0.074)} = 1350\ MW$

For correct substitution — 1 point

For correct answer — 1 point

$Q_{in} = Q_{out} + Q_{useful}$ — 1 point

$\frac{Q_{out}}{\Delta t} = \frac{Q_{in}}{\Delta t} - \frac{W_{useful}}{\Delta t} = 1350\ MW - 100\ MW = 1250\ MW$ — 1 point

Alternate Solution — (Alternate Points)

$e = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$

$\frac{Q_{out}}{Q_{in}} = 1 - e$

$Q_{in} = \frac{Q_{out}}{1 - e}$

$Q_{in} = \frac{W_{useful}}{e} = \frac{Q_{out}}{1 - e}$ — (3 points)

$\frac{W_{useful}/\Delta t}{e} = \frac{Q_{out}/\Delta t}{1 - e}$

$\frac{100\ MW}{0.074} = \frac{Q_{out}/\Delta t}{0.926}$ — (1 point)

$Q_{out}/\Delta t = 1250\ MW$ — (1 point)

If some attempt was made to use the efficiency from part (a) and the useful work, 1 point was awarded. If the student assumed the Carnot efficiency was 100% and then logically reasoned that $Q_{out} = 0$, 2 points were awarded.

5. (cont)

(c) 4 points

For recognition that power and energy are related by $P = \frac{E}{t}$ — 1 point

The energy needed in one second is

$E_{needed} = \frac{E_{useful}}{0.4} = \frac{(100\ MW)(1\ s)}{0.4} = 250\ MJ$ — 1 point

$E_{needed} = mc^2$ — 1 point

$m = (250 \times 10^6\ J)\ /\ (3 \times 10^8\ m/s)^2 = 2.77 \times 10^{-9}\ kg$ — 1 point

If student used the 40% efficiency combined with some energy in the problem, 1 point was awarded.

(d) 4 points

	Q	ΔT
AB	+	0
BC	0	–
CD	–	0
DA	0	+

4 points

1/2 point was awarded for each correct entry, and score was rounded down to nearest whole number when necessary.

HEAT ENGINES

1991

3.

(a) 3 points

For a correct expression for the efficiency in terms of energy or power input and output:

$e = \frac{P_{out}}{P_H}$ or equivalent — 1 point

(This point was not awarded for $e = 1 - \frac{P_C}{P_H}$ or equivalent with no further work, because it is not directly applicable using the given information.)

For substitution of $P_{out} = 120$ MW, $e = 0.4$ — 1 point

$P_H = 300$ MW — 1 point

1991 Physics B Solutions **Distribution of points**

3. (continued)

(b) 3 points
For an expression relating the mass to power or heat:

$\frac{\Delta m}{\Delta t} = \frac{P}{L}$ or $Q = mL$ — 1 point

For substituting P_H = answer from part (a) — 1 point

$\frac{\Delta m}{\Delta t} = 300\ \text{MW}/4.4 \times 10^7\ \text{J/kg}$

For correct answer:
$\frac{\Delta m}{\Delta t} = 6.82\ \text{kg/s}$ — 1 point

(c) 3 points
For an expression relating energy or power input and output:
$P_C = P_H - P_{out}$ (or equivalent) — 1 point

For substituting P_H = answer from (a) and P_{out} = 120 MW or equivalent — 1 point

$P_C = 300\ \text{MW} - 120\ \text{MW} = 180\ \text{MW}$ — 1 point

(d) 5 points
For correctly relating heat and temperature change:
$Q = mc\Delta T$ — 1 point

For recognizing that Q = (Power)(time) — 1 point

$\Delta T = Pt/mc$

For calculating the mass of the water:
$m = (1 \times 10^3\ \text{kg/m}^3)(200\ \text{m})(50\ \text{m})(10\ \text{m})$
$m = 1 \times 10^8\ \text{kg}$ — 1 point

For substituting $P = P_C$ from part (c) — 1 point

$$\Delta T = \frac{(180\ \text{MW})(1\ \text{hr})(60\ \text{min/hr})(60\ \text{s/min})}{(1 \times 10^8\ \text{kg})(4.2 \times 10^3\ \text{J/kg °C})}$$

$\Delta T = 1.5\ °\text{C}$ — 1 point

For at least one answer with correct units and no incorrect units — 1 point

Solutions	Distribution of points
(a) 3 points	
For any expression for the mechanical power	1 point
$P = Fv$ OR $\frac{W}{t}$ OR $\frac{\Delta K}{t}$	
For knowing that the force $F = mg$ (or $W = mgd$ in substituting for W, or $v^2 = 2gd$ in substituting for ΔK)	1 point
$P = mgv$	
$P = (10\ kg)(10\ m/s^2)(4\ m/s)$	
For the correct answer	1 point
$P = 400\ W$	
(b) 3 points	
The maximum possible efficiency is that of a Carnot cycle.	
For any recognition of this fact, expressed in words or by an attempt to write the appropriate equation	1 point
For the correct formula for the efficiency	1 point
$e_{max} = \frac{T_h - T_c}{T_h}$	
$e_{max} = \frac{500\ K - 300\ K}{500\ K}$	
For the correct answer	1 point
$e_{max} = 0.4$ or 40%	

Solutions	Distribution of points
(c)	
i. 2 points	
For an expression relating the rates of energy transfer to the efficiency	1 point
$e = \frac{W}{Q_h} = \frac{P}{\Delta Q_h/\Delta t}$	
Solving for the rate $\Delta Q_h/\Delta t$ at which the hot reservoir supplies heat:	
$\frac{\Delta Q_h}{\Delta t} = \frac{P}{e}$	
$\frac{\Delta Q_h}{\Delta t} = \frac{400\ W}{0.4}$	
For the correct answer	1 point
$\frac{\Delta Q_h}{\Delta t} = 1{,}000\ W$	
ii. 2 points	
For any indication of conservation of energy	1 point
$\frac{\Delta Q_c}{\Delta t} = \frac{\Delta Q_h}{\Delta t} - P$ OR $Q = \Delta U + W$ OR a heat engine diagram	
$\frac{\Delta Q_c}{\Delta t} = 1{,}000\ W - 400\ W$	
For the correct answer	1 point
$\frac{\Delta Q_c}{\Delta t} = 600\ W$	

1998 Q3

	Distribution of points
(a) 1 point	
For the correct value of work required to raise the center of mass of the lead $W = mgh$	1 point
(b) 3 points	
For a correct expression for the change in gravitational potential energy, mgh	1 point
For a correct expression for the change in thermal energy, $mc\Delta T$ This point was also awarded for using the expression $nc_V \Delta T$, but in this case the final point was <u>not</u> awarded	1 point
Equating the two energy changes $mgh = mc\Delta T$	
For the correct answer $\Delta T = \frac{gh}{c}$	1point
(c) 2 points	
For correctly substituting values in the answer to part (b) and a correct calculation	1 point
For multiplying that value by 100	1 point

$$\Delta T_{cum} = 100\frac{(9.8\ \text{m/s}^2)(2.00\ \text{m})}{128\ \text{J/kg}\bullet\text{K}}$$

$\Delta T_{cum} = 15.3\ \text{K}$ (15.6 K when using $g = 10\ \text{m/s}^2$.)

(d) 2 points	
The answers to parts (b) and (c) will not change, because ΔT does not depend on the mass.	
For correctly saying “no” and including a correct physical explanation	2 points
Full credit was awarded for “yes” plus an explanation if this was consistent with student’s answers to (b) and (c).	

	Distribution of points
Question 3 (continued)	
(e) 2 points	
For any completely correct answer referring to energy transfer For example: There is friction between the lead and the apparatus, resulting in a loss of heat to the surroundings.	2 points
Only one point was awarded for an answer that had a correct reference to energy transfer but also had incorrect statements.	
Only one point was awarded for a correct answer without elaboration, for example, “Friction”, “Air resistance”, or “Time elapsed”	