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Special Focus: Electrostatics

A Note from the Editor

Peggy Bertrand
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One of the most difficult areas of physics to learn, and therefore to teach, is electrostatics, and this is largely due to the highly abstract nature of the topic. You just can't see excess electrons, and it is even harder to see their *absence*! And, it is *supremely* difficult to visualize the modification of empty space by a configuration of stationary charges, and to understand how this modification of space will affect other charges that might be placed there. Students who had little difficulty with Newtonian mechanics often have a devil of a time visualizing electric forces, fields, potential energies, and potential surfaces, despite our best efforts to help them understand. Many of us will remember the frustration we felt during our first exposure to electrostatics—I know I do! As a result, I am always eager to adopt new teaching strategies from my colleagues, some of whom have very creative ideas that I have found effective when I've tried them out with my own students.

The first article of this series, "Basic Concepts in Electrostatics: An Overview," by Hasan Fakhruddin, serves as a "jump-start" content introduction to electrostatics that will help new teachers come up to speed on the concepts quickly. This article is at the Physics B level, but a brief Physics C extension appears at the end. The next article is the instructional unit "Modern-Day Faradays: Teaching Students to Visualize Electric Fields." In it, Marc Reif discusses strategies for helping students in either Physics B or Physics C visualize and draw electric fields. Next, Connie Wells's instructional unit on "Electric Potential and Potential Energy" incorporates plenty of activities to help Physics B and Physics C students see and understand the difficult concepts of potential surfaces and the associated potential energies of charged particles located in electric fields. The next instructional unit, "Teaching Gauss's Law," by Martha Lietz, incorporates some creative approaches and hands-on activities for teaching the highly abstract topic of Gauss's Law to Physics C students. Finally, my article "Conceptual Links in Electrostatics" shamelessly exploits the students' desire to memorize equations to trick them into focusing on the physical links between force, field, potential, and potential energy, and contains material for Physics B and Physics C. Each one of these articles comes complete with assessment questions, practice problems, and of course the solutions.

As a personal aside, let me state that as I read the material submitted by my colleagues, I was very impressed. I decided to try out some of the activities presented in these articles with my own students—and they work! On behalf of all the contributors, I would like to express our sincere hope that you and your students find these articles valuable as you embark on the difficult task of teaching electrostatics.

Basic Concepts in Electrostatics: An Overview

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History

The early Greek, Thales of Melitus, discovered that when a piece of amber ("electron" in Greek) was rubbed with wool, it could attract bits of wool and straw. Benjamin Franklin discovered that the electric charges are of two kinds, and he arbitrarily named them positive and negative. He called the charge that appeared on a glass rod when it was rubbed with silk, positive, and the charge on an ebonite rod that was rubbed with fur, negative.

Electric Charges

In a simplified model of the atom, its tiny nucleus contains two kinds of particles: positively charged *protons* and neutral *neutrons*. There are negatively charged *electrons* orbiting the nucleus in some specific orbitals. Proton and electron have equal magnitude and opposite sign charges. Hence, in the neutral atom, there are as many protons in the nucleus as there are electrons moving around the nucleus. All substances are ultimately made up of atoms of one or more kinds. By mechanical actions such as rubbing two objects together, electrons can be transferred from one object to the other; the protons that are tightly bound in nuclei are not transferred. Due to the nature of atoms or molecules making up a substance, it may accept electrons or give up electrons when rubbed with another substance. For example, when silk and glass are rubbed together (or even just brought into physical contact), silk gains electrons from glass because it has a greater affinity for electrons. Hence, silk having excess electrons is negatively charged, and glass having a deficiency of electrons is positively charged.

Quantization of Charges

The proton and electron carry the smallest possible stable amount of positive and negative charges; i.e., any stable charge is a multiple of the charge on a proton or electron. In SI units:

$-e$ = charge on an electron = -1.6×10^{-19} coulomb
e = charge on a proton = $+1.6 \times 10^{-19}$ coulomb
where the coulomb is the SI unit of charge

Conservation of Charges

In any process, electric charge is conserved. Neutralization of an object only means equalization of positive and negative charges. It is not possible to create just negative or just positive charges. Under ordinary conditions, charging involves transfer of negatively charged electrons from one object to another.

Interaction Between Electric Charges

Like charges repel, and unlike charges attract. The forces of attraction (or repulsion) act on two interacting charged particles and are equal in magnitude and opposite in direction, even if the charges are unequal in magnitude (as dictated by Newton's Third law of Motion).

Charging by Conduction and Induction

Under ordinary conditions, an object can be charged by *conduction* or *induction*.

Charging by Conduction: A charged object, when brought into physical contact with a second object, may share some or all of its charges with the second object. Thus the second object gets the same type of charge as the first object originally had.

Charging by Induction: When a charged object is brought close to a neutral object, charge migration in the neutral object can occur, causing it to assume an opposite charge near the charging object and a like charge far away from the charging object. Thus the second object gets electrically polarized. The two charges on the second object are equal and opposite, and on the whole the object still has net zero charge. The induced opposite charge on the second object closer to the charging object is called *bound charge* and the farther like charge is called *free charge.* The free charge can be removed by providing a conducting path to ground, achieved by touching the polarized object, provided the charge object is held in place during the process.

Charged Conductors and Charged Insulators

There are important differences between a charged conductor (most metals) and a charged dielectric material (insulators like plastics, mica, glass, oils, etc.)
Under steady state conditions:

- Any charge given to a conductor resides on its outer surface, and charge density is greater at sharper regions on the surface.
- Any charge given to an insulator stays in place. Hence, a plastic object can have charge spread throughout its volume; this is not possible for a conducting object where the charge placed inside the conductor will move to its outer surface.

Coulomb's Law

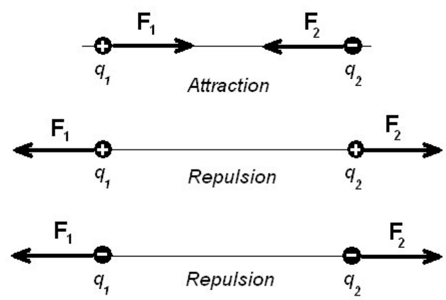
Charles Coulomb [1736–1806], through his experiments, formulated the law governing the interaction between electric charges.

According to this law, for two charged particles q_1 and q_2 a distance r apart (see the figure above), the electric force of attraction or repulsion between them is directly proportional to the product of the charges and inversely proportional to the square of the distance r between them. Thus, the magnitude of the force is given by:

$$F = \frac{k_e\,|q_1 q_2|}{r^2}$$

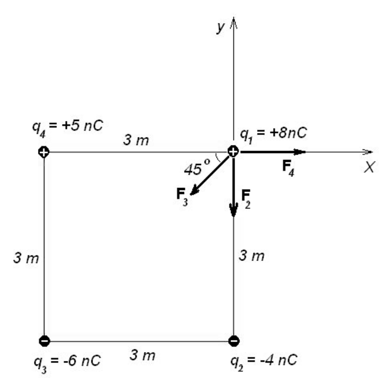
where $k_e = 9.0 \times 10^9 \,\frac{Nm^2}{C^2}$. The constant k_e is also written as $\frac{1}{4\pi\varepsilon_0}$ where $\varepsilon_0 = 8.85 \times 10^{-12}\,\frac{C^2}{Nm^2}$ and is called the *electrical permittivity* of a vacuum.

The direction of the forces on each particle depends on the sign of the charges, as illustrated in the figures below:



In all the cases above, the forces $\mathbf{F_1}$ and $\mathbf{F_2}$ are equal in magnitudes and opposite in directions; i.e., $\mathbf{F_1} = -\mathbf{F_2}$ and $F_1 = F_2$.

It must be remembered that forces are vectors, and therefore have magnitude and direction. If two charges are both exerting forces on a third charge, it may be necessary to find the net force through vector addition. The example below illustrates how to calculate the net force on the charge q_1 due to the other three charges.



As shown in the figure above, four point charges, q_1, q_2, q_3, and q_4, are placed at the four corners of a square of side 3.0 m. If $\mathbf{R}$ is the resultant force, then $\mathbf{R} = \mathbf{F_2} + \mathbf{F_3} + \mathbf{F_4}$.

$$F_2 = \frac{k_e |q_1 q_2|}{(3.0m)^2} = \frac{\left(9 \times 10^9 \frac{Nm^2}{C^2}\right) |(8.0 \times 10^{-9}\ C)(-4.0 \times 10^{-9}\ C)|}{(3.0m)^2} = 3.2 \times 10^{-8}\ N$$

$$F_3 = \frac{k_e |q_1 q_3|}{(3\sqrt{2}m)^2} = \frac{\left(9 \times 10^9 \frac{Nm^2}{C^2}\right) |(8.0 \times 10^{-9}\ C)(-6.0 \times 10^{-9}\ C)|}{(3\sqrt{2}m)^2} = 2.4 \times 10^{-8}\ N$$

$$F_4 = \frac{k_e |q_1 q_4|}{(3.0m)^2} = \frac{\left(9 \times 10^9 \frac{Nm^2}{C^2}\right) |(8.0 \times 10^{-9}\ C)(-5.0 \times 10^{-9}\ C)|}{(3.0m)^2} = 4.0 \times 10^{-8}\ N$$

Use the *component method* to add the forces together to obtain the resultant force. The angles for the three forces are measured counterclockwise from the positive x-axis for the purpose of finding their x and y components below.

$$R_x = F_{2x} + F_{3x} + F_{4x} = F_2\ cos(270°) + F_2\ cos(225°) + F_2\ cos(0°)$$
$$= 0\ N - 1.697 \times 10^{-8}\ N + 4.0 \times 10^{-8}\ N = 2.303 \times 10^{-8}\ N$$
$$R_y = F_{2y} + F_{3y} + F_{4y} = F_2\ sin(270°) + F_2\ sin(225°) + F_2\ sin(0°)$$
$$= -3.2 \times 10^{-8}\ N - 1.697 \times 10^{-8}\ N + 0 = -4.897 \times 10^{-8}\ N$$

$$R = \sqrt{R_x^2 + R_y^2} = 5.41 \times 10^{-8}\ N$$
$$\theta = tan^{-1}\left|\frac{R_y}{R_x}\right| = 64.8°;\ 3^{rd}\ \text{quadrant}$$

Electric Field and Electric Field Lines

The *electric field* is a way of describing how a point charge or a distribution of discrete or continuous charges influences the space around them. The electric field at various points around an electric charge distribution can be analyzed by measuring the force the distribution creates on a *test charge*. The *test charge* is a positive point charge of unit magnitude. It is assumed that the test charge does not disturb the charge distribution or create a significant field of its own.

The *electric field lines* (also called *electric lines of force*) present a visual illustration of the electric field. The electric field lines can be imagined to be traced by the test charge released in the electric field when allowed to move slowly as if in a viscous medium. Below are some important properties of electric field lines:

Properties of Electric Field Lines

- A test charge will trace out an electric line of force if allowed to move slowly in the field.
- At any point, a tangent drawn to the line of force represents the direction of electric field at that point.

- Any number of lines can be drawn in a region; however, the number of field lines should be proportional to the magnitude of the electric field in that region of space.
- The lines of force are crowded in regions of high intensity electric field and are spaced farther apart in regions of low intensity electric field.
- No two lines of force intersect each other.
- In a uniform electric field, the lines of force are parallel and evenly spaced.
- The lines of force originate at the positive charges and terminate at the negative charges.
- The lines of force start at right angles to the surface of a charged conductor.
- No electric field lines are present inside a conductor under steady state conditions.

Quantifying Electric Field Strength (Intensity)

Electric field strength (intensity), E, at a point in an electric field can be quantified by defining it to be *the electric force experienced by a charge of* 1 *coulomb placed at that point*.

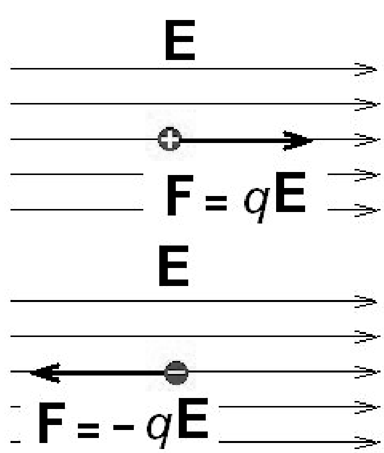
Thus, if any charge q is placed at that point it will experience a force **F** given by:

$$\mathbf{F} = q\mathbf{E}$$

Direction of E at a point: Since the test charge is positive, the direction of **E** at any point is the same as the direction of the electric force on a positive charge placed at that point.

Direction of F on a charged particle at a point in an electric field: If a charged particle q is placed in a region where the electric field is **E** (see the figure below), then the following statements can be made:

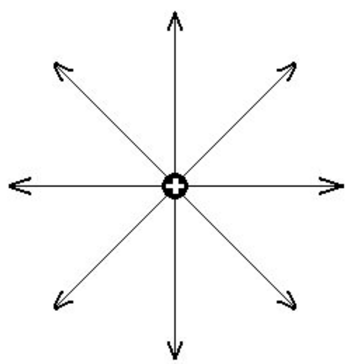
- The direction of **F** is the same as the direction of **E** if q is positive.
- The direction of **F** is opposite to the direction of **E** if q is negative.



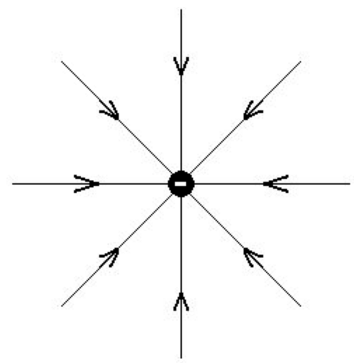
Electric Field Due to a Point Charge:

Following the definition of electric field strength **E**, an expression for **E** due to a point charge q can be derived. The result is:

- The magnitude of **E** is given by $E = \frac{k_e|q|}{r^2}$
- The direction of **E** is away from q if q is positive and toward q if q is negative.



Electric field due to a positive point charge at any point is away from the charge



Electric field due to a negative point charge at any point is toward the charge

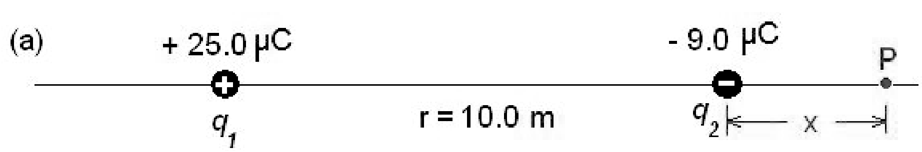
The Electric Field Due to a Charge Configuration: Superposition

The electric fields due to individual charges in a charge configuration must be added together to determine the net electric field at a given point. This phenomenon is often referred to as *superposition*.

Superposition can be used to locate the point(s) at which $\mathbf{E}_{net} = 0$ for each of the pairs of fixed charges below:

For the cases below, $\mathbf{E}_{net}$ cannot be zero at any point off the line because the fields $\mathbf{E}_1$ due to q_1 and $\mathbf{E}_2$ due to q_2 at that point will not be collinear and hence cannot cancel each other out.

***Case* (*a*):** Determine the position on the x-axis in which the net electric field, $\mathbf{E}_{net}$, is zero.



For the following locations $\mathbf{E}_{net}$ cannot be zero

- Between q_1 and q_2, because $\mathbf{E}_1$ and $\mathbf{E}_2$ both point to the right.
- To the left of q_1, because though $\mathbf{E}_1$ and $\mathbf{E}_2$ point in opposite directions, $E_1 > E_2$.

Let $\mathbf{E}_{\mathbf{net}} = 0$ at a point P, a distance x to the right of q_2. $\mathbf{E}_1$ and $\mathbf{E}_2$ point in opposite directions, hence $E_1 = E_2$.

$$\frac{k_e|q_1|}{(10.0m + x)^2} = \frac{k_e|q_2|}{x^2}$$

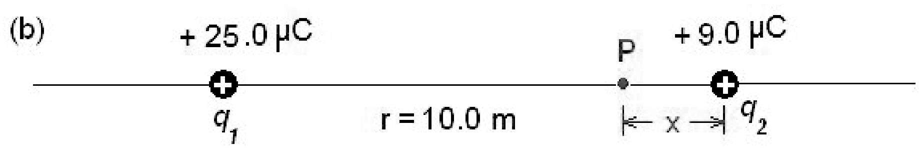
$$\frac{|25.0 \times 10^{-6}\,C|}{(10.0m + x)^2} = \frac{|-9.0 \times 10^{-6}|}{x^2}$$

$$x = 15m, -3.75m$$

$$\therefore\ x = 15m$$

which is to the right of q_2.

***Case* (*b*):** Determine the position on the x-axis in which the net electric field, $\mathbf{E}_{\mathbf{net}}$, is zero.



For this case, $\mathbf{E}_{\mathbf{net}}$ cannot be zero to the left of q_1 and to the right of q_2, because in those regions, $\mathbf{E}_1$ and $\mathbf{E}_2$ both point in the same direction. Let $\mathbf{E}_{\mathbf{net}} = 0$ at a point P between q_1 and q_2 at a distance x to the left of q_2. $\mathbf{E}_1$ and $\mathbf{E}_2$ point in opposite directions, hence $E_1 = E_2$.

$$\frac{k_e|q_1|}{(10.0m - x)^2} = \frac{k_e|q_2|}{x^2}$$

$$\frac{|25.0 \times 10^{-6}\,C|}{(10.0m - x)^2} = \frac{|+9.0 \times 10^{-6}|}{x^2}$$

$$x = -15m, 3.75m$$

$$\therefore\ x = 3.75m$$

which is to the left of q_2.

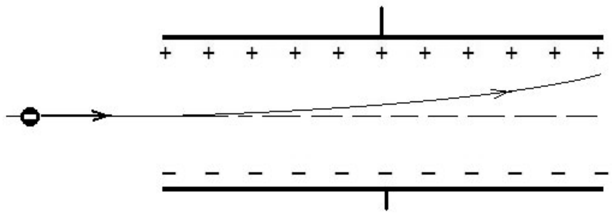
Motion of a Charged Particle in Electric Field

The acceleration of a particle of charge q and mass m in an electric field $\mathbf{E}$ is given by

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q\mathbf{E}}{m}$$

- A *positively* charged particle accelerates in the *same direction* as the applied electric field **E.**
- A *negatively* charged particle accelerates in the *opposite direction* to the applied electric field **E.**

A more complicated example involves deflection of a charged particle in the uniform E-field between two parallel plates as shown on the following page.



In the figure, the electric field is directed from the positive top plate to the negative bottom plate and is of uniform intensity. The negatively charged particle experiences an electric force in the opposite direction of this field. It retains its velocity in the x-direction when it enters the field, and accelerates in the $+y$-direction. The resulting path of the particle through the field is shown.

Students should realize that this problem is very similar to the problems on projectile motion where an object is launched horizontally from a point above ground.

Electric Potential Energy

The electric potential energy, U_E, between two charged particles, q_1 and q_2, is given by:

$$U_E = \frac{k_e q_1 q_2}{r}$$

This equation assumes that if the charged particles are at an infinite distance apart, the potential energy of the system is defined to be zero.

Electric Potential (V)

The electric potential at any point is defined as the work done per unit charge in bringing a positive point charge q from infinity to that point. Thus,

$$V_P = \frac{W_{\infty,P}}{q}$$

The electric potential energy of a point charge q at a point in space with potential V is:

$$U_E = qV$$

The work done by an external force in moving a charge from point 1 to point 2 is given by:

$$W = q(V_2 - V_1)$$

If a point charge, q, subjected only to the electric force moves from point 1 at potential V_1 to a point 2 at potential V_2 then:

$$KE_1 + qV_1 = KE_2 + qV_2$$
$$\Delta KE = q(V_1 - V_2)$$

A Useful Equation: If a charged particle with charge q and mass m is accelerated from rest through an accelerating potential of V volts, then:

$$qV = \frac{1}{2}mv^2$$

This equation is useful in determining the speeds of charged particles in mass spectrometers, and the speed of electrons in an electron microscope. In such cases, the initial speeds of the charged particles are assumed negligible.

Potential due to a point charge:
Let P be a point a distance r from a point charge q. The potential at P due to the charge q is:

$$V = \frac{k_e q}{r}$$

The electric potential V is a scalar quantity. It can be positive, negative, or zero.

Net potential due to two or more point charges:
If there are a number of point charges q_1, q_2, q_3 … a distance r_1, r_2, r_3 … from a point P, then the net potential V at P is the sum of the potentials V_1, V_2, V_3 … due to all the point charges. Thus,

$$V = V_1 + V_2 + V_3 \cdots$$

or

$$V = \frac{k_e q_1}{r_1} + \frac{k_e q_2}{r_2} + \frac{k_e q_3}{r_3} + \ldots$$

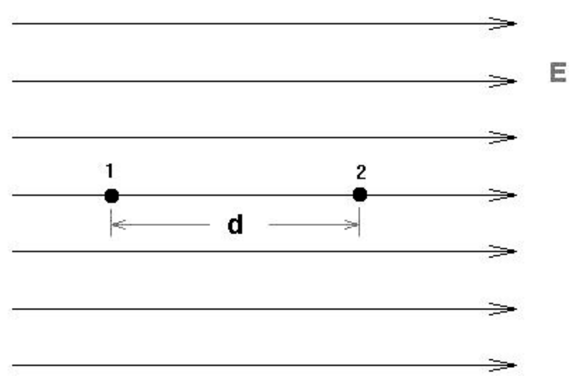
Potential Due to a Charged Spherical Conductor:
For a charged spherical conductor (hollow or solid) of radius R, the electric potential at its surface is the same as if the entire charge of the sphere is concentrated at the center of the sphere. Hence,

$$V \text{ (at the surface of a charged sphere)} = \frac{k_e q}{R}$$

However, inside the sphere the electric potential is constant for all the points and is the same as that at the surface. Hence,

$$V(\text{surface}) = V(\text{at any point inside})$$

Potential difference between two points in a uniform electric field E:
For the points 1 and 2 in a uniform electric field of magnitude E as shown in the figure below



$$V_1 - V_2 = Ed$$

Also note that the point 1 is at a higher potential compared to the point 2. Why?

Emphasize for students that electric potential decreases as one moves in the direction of electric field. Thus in the diagram above, $V_2 < V_1$.

Electric field from electric from electric potential:
As in the above diagram, if points 1 and 2 are on a field line in a uniform electric field **E** and potential difference between the points is

$$V_1 - V_2 = V$$

then the magnitude of the electric field can be calculated from the equation

$$E = \frac{V}{d}$$

This equation also gives us the more commonly used unit for **E**; i.e., volts/meter.

Students should know which of the two equations $E = \frac{k_e|q|}{r^2}$ ***or*** $E = \frac{V}{d}$***, they should use to determine the magnitude of E in a problem.***

Electron Volt:
An 'electron volt' (eV) is a unit of energy.

One eV is the kinetic energy gained by a particle carrying an elementary charge $e = 1.6 \times 10^{-19}\ C$, accelerated through a potential difference of 1 *volt*. Thus

$$1\ eV = q\,\Delta V = (1.6 \times 10^{-19}\ C)\,(1\ V) = 1.6 \times 10^{-19}\ J$$
$$1\ keV = 1{,}000\ eV$$
$$1\ MeV = 1{,}000{,}000\ eV$$
$$1\ GeV = 1{,}000{,}000{,}000\ eV$$

For example:

- An electron accelerated through a potential difference of 1 V will gain 1 eV of kinetic energy.
- A proton accelerated through a potential difference of 1 V will also gain 1 eV of kinetic energy, but will be accelerated in the opposite direction as the electron.
- A proton accelerated through a potential difference of 1×10^6 *volt* will gain 1 MeV of kinetic energy.

A particle carrying 3 times the elementary charge accelerated through a potential difference of 1 V will gain 3 eV of energy.

Equipotential Surfaces:

- In an electric field, an equipotential surface is one which has all its points at the same potential.
- The electric field vectors are perpendicular to the equipotential surfaces.
- For a point charge, the equipotential surfaces are concentric spherical surfaces with the point charge at the center. The electric field, being radial, is always perpendicular to these surfaces.
- For a uniform field in which the field lines are parallel and equally spaced, the equipotential surfaces are the planes intersecting the field lines at right angles.
- If a uniform electric field is created between two parallel plates, the equipotential surfaces are planes parallel to the plates.

For Physics C: An Extension

Obtain an expression for the potential energy, U_E, for two charged particles, q_1 and q_2, a distance r apart, using Conservation of Mechanical Energy and calculus. Let q_1 be at a point P. The particle q_2 is placed a distance r from it. Now move the particle to a distance r'. Thus

$$U_E' - U_E = -\int_r^{r'} \mathbf{F} \cdot d\mathbf{r} = -\int_r^{r'} F dr = -\int_r^{r'} \frac{k_e q_1 q_2}{r^2} dr$$
$$= k_e q_1 q_2 \left(\frac{1}{r'} - \frac{1}{r}\right)$$

When $r' = \infty$, $U_E{}' = U_\infty = 0$:

$$U_E = \frac{k_e q_1 q_2}{r}$$

The difference of electric potential between two points A and B in an electric field is defined as:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

In the field of a point charge q, this integral yields

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

To define the *electric potential* at point A, point B is assumed to be at infinity. Hence the above equation becomes:

$$V_A = \frac{k_e q}{r_A}$$

If a point charge q is moved from position 1 to another position 2, its change in potential energy is defined as follows:

$$\Delta U_E = \int_1^2 \mathbf{F} \cdot d\mathbf{r} \ \text{ if } \mathbf{F} \text{ is } \textit{external} \text{ force moving the charge q}$$

$$\Delta U_E = -\int_1^2 \mathbf{F} \cdot d\mathbf{r} \ \text{ if } \mathbf{F} \text{ is the } \textit{electric} \text{ force moving the charge q}$$

As shown above, the potential difference between points 1 and 2 is given by:

$$\Delta V = -\int_1^2 \mathbf{E} \cdot d\mathbf{r}$$

If V is a function of the coordinates x, y, and z, then the x-, y-, and z-components of electric field at any point P(x,y,z) is given by:

$$\mathbf{E} = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\hat{\mathbf{j}} - \frac{\partial V}{\partial z}\hat{\mathbf{k}}$$

Modern-Day Faradays: Teaching Students to Visualize Electric Fields

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Introduction

Set the Stage for E-field in Mechanics

What is mass and what is charge? Physical quantities are difficult to define to the curious student's satisfaction. The teacher who attempts to answer the question "But what is it really?" risks slipping into metaphysics. Teachers will likely find it safest to confine themselves to operational definitions of these slippery concepts. Here is a reasonable working definition of mass: mass is that aspect of matter which causes it to resist changes in motion ("inertial mass") and to exhibit a gravitational force ("gravitational mass"). Or, even simpler: mass is a comparison of an object to a standard mass, using a balance. The presentation of these concepts is made easier because these properties of mass can be observed and felt directly by students.

Students will benefit from a clear and concise definition of charge made explicit from the start. This could proceed from a simple demonstration: charge a balloon or plastic rod by rubbing it on fur and use it to pick up small pieces of paper or make students' hair stand on end. The question "Why does the paper/hair behave like this?" leads quickly to an operational definition for charge: Charge is the aspect of matter that causes matter to exhibit an electromagnetic force.

Force is a (the?) Key Concept in Both Introductory Mechanics and Electrostatics

The logical starting point for an investigation of the electric field is perhaps its most concrete manifestation, the electrostatic, or Coulomb force. Students should have some qualitative hands-on experiences with electric forces before they are asked to draw electric field maps. Part of the fun of teaching electrostatics is that students may easily recreate the great discoveries of an earlier age. Some time spent in relatively unstructured "playing" with charged objects gives students a feel for the behavior of the electrostatic force. Useful activities could include working with charged rods in stirrups, charged balloons, or "sticky tape" experiments.[1,2] Materials for these activities can be purchased relatively inexpensively from science supply vendors, or household items purchased at a grocery store or Wal-Mart may be substituted. Plastic golf tubes rubbed with oven roasting bags are a convenient source of a fairly large (negative) charge. A useful technique is to encourage students to visualize what is happening by drawing simple diagrams of the objects involved with "pluses" and "minuses" to represent the charges on the objects. Applets and simulations such as the

PhET simulation of a charged balloon[3] can help students understand what is happening by providing a "microscopic mental picture."

Introducing the concept of field earlier in the course with a familiar and less-complicated example helps prepare students for the more involved electric field diagrams and potential maps they will encounter in electrostatics. Many teachers begin teaching the field concept when they discuss gravitation. Universal gravitation (for point masses) is analogous to the Coulomb force (for point charges). The magnitudes of the forces follow similar equations:

$$F_G = \frac{Gm_1m_2}{r^2} \qquad\qquad F_E = \frac{k|q_1||q_2|}{r^2}$$

Both are inverse-squared relationships with "odd constants." Both depend on the product of a conserved quantity (mass in the case of gravity, charge in the case of the electrostatic force). And the constants for both were originally measured with an elegantly simple torsion balance.

The Electric Field

Most of the forces encountered in introductory physics are "contact forces." In other words, objects must be touching to influence each other. The troubling aspect of both the gravitational and electric forces is the way that they are able to reach across "empty space" and influence other objects, as "non-contact forces." This "action-at-a-distance" was said to have troubled Newton, and he offered no explanation for it, famously writing near the end of *The Principia*: "I frame no hypotheses."[4] Well, if it troubled Newton, you can bet that it will trouble some of your students. The field concept is among other things a way to conceive of what is happening in "action-at-a-distance." A gravitational field is created by an object with mass, and an object with charge creates an electrical field.

An object with mass sets up a gravitational field, which is a characteristic of space. We can imagine the field to be consisting of lines of force arranged three-dimensionally around the object. Since the gravitational force is always attractive, all of the lines end in an arrowhead on the object in question. Another object with mass, a small "test mass," placed on one of those lines would feel a force on it in the direction of the line and pointing toward the object from which the field originates. The strength of the field is proportional to the number of lines present in that region of space. "But," the student objects, "we imagined it, so it can't be real." And they've got a good point. However, the fact remains that the "imaginary" field can be measured. Moreover, the field concept is a very useful one for explaining and predicting the behavior of real objects, so we must go with it.

The electric field differs from the gravitational field in having two possible directions. There are two types of charge, positive and negative. By definition, electric field originates on positive charge and ends on negative charge. The field is defined for a positive "test charge."

Placed on a field line, the force on the positive test charge would be directed away from the positive charge and towards the negative charge.

Gravitational and electric fields for point masses (the spherical earth behaves as a point mass) and point charges are analogs. The magnitude equations for these fields appear below. The field direction is given by the direction of the force on a test mass for the gravitational field or on a test charge for the electric field.

$$g = \frac{GM}{r^2} \qquad E = \frac{kq}{r^2}$$

In the presence of a constant field, we also write simpler analogous force equations:

$$\mathbf{F}_G = m\mathbf{g} \qquad \mathbf{F}_E = q\mathbf{E}$$

For a uniform field, the field strength is a ratio

$$\mathbf{g} = \frac{\mathbf{F}_G}{m} \qquad \mathbf{E} = \frac{\mathbf{F}_E}{q}$$

A field can be defined as any physical quantity that can be measured throughout space. Students can more easily recognize the concept of the field when it begins with gravitation. A handheld spring scale with a lab mass on the end can be moved around in the classroom, demonstrating the practical constancy of the gravitational field. Asking students to experimentally determine the ratio of gravitational mass to force in the classroom can be an eye-opener for many. Although the answer may seem obvious to the teacher, it will not be obvious to many of the students, and most set about the task without a definite expectation that a graph with a slope of approximately 9.81 N/kg will be found. It is also helpful at this point to clearly distinguish between the force from the earth/mass ratio in a static situation and the gravitational acceleration by using "N/kg" for the former and "m/s^2" for the latter. Asking students to produce field maps for the gravitational field visualized from an observer on the surface of the earth (*Figure* 1) and for an observer looking at the earth from space (*Figure* 2) can be a useful precursor to the more elaborate field maps that students will be asked to draw in electrostatics. The gravitational field in the classroom can be illustrated by arrows cut from cardboard or plastic, regularly spaced, and hanging down from the ceiling by strings. Each arrow can be scaled or labeled $g = 9.81$ N/kg.

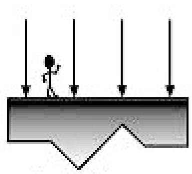


Figure 1

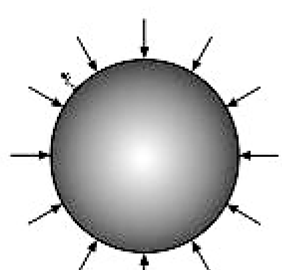


Figure 2

As in gravitation, the forces in electrostatics can be easily experienced in the classroom. Charged balloons, sticky tapes, and Van de Graaff generators can be used to demonstrate the potential of excess electrons to change the motion of objects. Michael Faraday, who initially developed the concept of the electric field, defined the field's vector direction using a small positively charged object, or "test charge," on an insulated stand.[5] Thus, the field for an isolated positive point charge (or a spherical object) and an isolated negative point charge are visualized below (*Figure* 3):

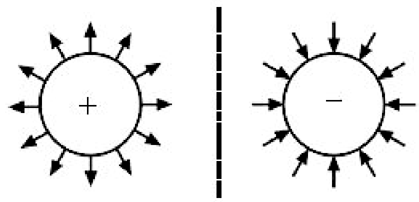


Figure 3.

After students have a feel for what is happening they should be asked to sketch simple electric field plots. It is often more effective to ask them to attempt this task before they have viewed textbook, online, or video field representations. Remind students of what they have learned about the electrostatic force by playing with charge: it falls off rapidly with distance, and there are two kinds of charge. Begin by having students draw vectors representing the force on a test charge for several points around a point charge or arrangement of point charges. When they have correctly indicated the direction and relative magnitude for various positions, you may explain the purpose of the field map: to describe the direction and relative magnitude of the force on a test charge placed anywhere in the vicinity of a configuration of other charges. The rules governing field lines are:

1. Fields begin on positive charges, and end on negative charges.
2. The number of field lines in an area is proportional to the strength of the field.
3. Field lines never cross.
4. The electric field is always perpendicular to a conducting surface.
5. There is no electric field inside a conductor. (If there were, charge would be in motion inside the conductor, and this would not be electrostatics!)

On the AP® Physics B Exam, students may be asked to draw or interpret electric field maps for simple geometric arrangements of point charges. On the Physics C Exam, they may also be asked to work with charge distributions. In order to draw a field map for a collection of point charges, it helps if students keep in mind two facts: field lines leave a point charge radially, and very far from a collection of point charges with a net charge, the collection looks like a single point charge (no matter what it looks like up close). The University Physics II Course Guidebook from the University of Arkansas[6] offers a complete description of the method for drawing field maps:

- "Draw the Location and Strength of the Charges: Leaving plenty of room, draw circles where the electric charges are located. Label each circle with the strength of the charge. Select a Number of Lines Per Charge: The number of field lines entering or leaving a charged object is proportional to the charge of the object. If we have point charges $q_1 = 5\mu C$, and $q_2 = -10\mu C$, I might randomly select four lines to represent q_1, therefore eight lines represent q_2.

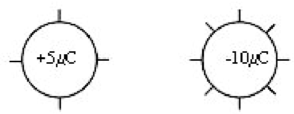


Figure 4

- Draw Stubs of Field Lines: Draw little arrows on the charges for the number of lines selected. Arrows should point out for positive charge and in for negative. Near a point charge the field lines are radial, since $E \to \infty$ as $r \to 0$. This means that the field line stubs should be evenly spaced around the charge.

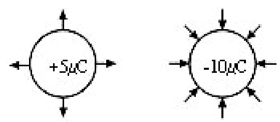


Figure 5

- Draw the Long Range Field: Far from a charge distribution, the electric field will have a characteristic shape. For distributions with a non-zero net charge, the electric field far from the charges will be that of a point charge with a charge equal to the total charge of the distribution. If we continue with q_1 and q_2 above, far from the charge we will see a radial field equal to that of a point charge with charge $q_t = q_1 + q_2 = -5$.
- Draw a dashed circle far from the charges, which is called the circle at infinity.
- Draw the appropriate number of field lines leaving or entering this circle.
  For q_1 and q_2, if four lines leave $q_1 = 5$, then four other lines must enter the circle at infinity since $q_t = -5$

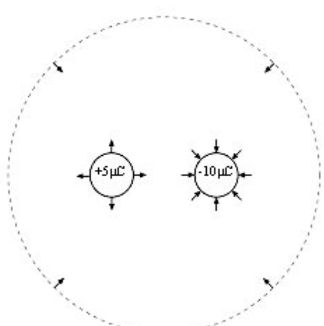


Figure 6

- Connect the Stubs Without Crossing the Field Lines: Field lines do not cross, since the electric field has a single direction at every point in space. (As Egon said, "Do not cross the streams … It would be bad.") To finish the map, simply connect the field lines on the stubs and the field lines at infinity, without crossing the lines. A line may not begin and end with stubs pointing in different directions.

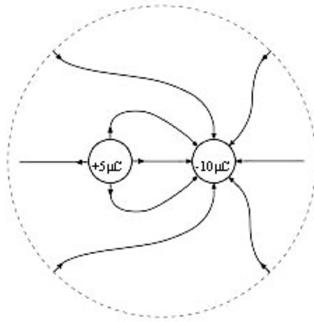


Figure 7. Author's Note: We would expect a point to the left of the positive charge where the field was zero. This diagram does not reflect that because of the orientation of the stubs. This diagram was included because this is a frequent occurrence using this method of drawing field maps.

- Respect Symmetry: The symmetry of your field map is affected by the initial choice of stub directions, your choices for the field at infinity, and how you connect the stubs. The field you end up with should have the same symmetry as the charges you started with."[6]

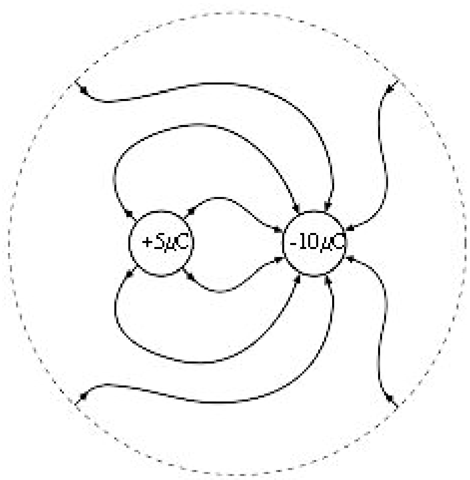


Figure 8. The corrected drawing with the stubs rotated.

Don't underestimate the value of students attempting to draw field maps by hand before they have looked at completed examples. If they see the examples beforehand, many will attempt to memorize and copy. If they try to draw them before seeing examples, they will have to draw upon what they know about force, vectors, and representation. Their experience with representations of gravitational field should make the task of representing the field from a point charge easier. The process of drawing the field from a collection of point charges is an excellent exercise in spatial reasoning.

It is often helpful to include multiple conceptual exercises when teaching fields. These may involve reasoning from collections of charged objects in space, or from field maps themselves. If done before instruction, these may serve as formative assessment, highlighting weaknesses in student understanding. In any case, they may further students' conceptual understanding. One source of these exercises is the book *Ranking Task Exercises in Physics.*[7] Each exercise contains multiple diagrams of situations that students, individually or in groups, must rank from greatest to least. A number of these deal with electric force, electric field, and electric potential.

Classroom Activities Using Technology

Software is a powerful tool for visualizing maps of electric fields. Programmed simulations can make electric field visualizations effortless. These may be used in a structured way, with students given a list of situations to create and questions to answer. Students may also be able to determine (or check) the answers to homework problems. It is possible to use the simulations in a more unstructured way, by posing open-ended questions to be answered using the simulations. This often works well early in a unit. For example, you might ask students "Is there any way to arrange X number of charges so that the electric field is zero somewhere?"

Both free Web-based and commercial products are available. A Web search using terms like "electric field" and including one of these search terms: "physlet," "applet," or "simulation," will turn up Web sites featuring interactive simulations that students can "play" with.[8]

An older commercial product, *EM Field*[9], allows students to arrange charges and view field and potential maps. A free Web-based *Charges and Fields* version is available from the PhET Web site.[3] Both *EM Field* and *Charges and Fields* allow students to place charges in their own arrangements in a plane and view the resulting electric field plot or potential map. Charge distributions may be represented by placing many point charges in succession. PhET's *Electric Field of Dreams* allows the charges to move (constrained within walls), and the application of an external field. Another older commercial product, *Electric Field Hockey*[9], gives students an entertaining game to play with electric field. The PhET Web site also contains a free version. The PhET Web site allows you to download one or all of their simulations to your local computer.

Microsoft Excel spreadsheets can be set up to produce two- and three-dimensional plots of electric charge arrangements.[10] In one popular experiment, students measure voltages across an arrangement of charged objects and enter the voltages into a spreadsheet. The spreadsheet plots equipotential, and students reason "backward" from equipotential to draw field lines by hand on the equipotential plots.

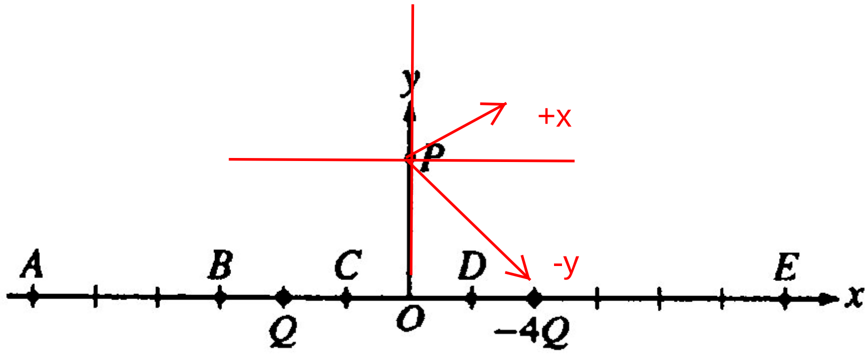
"The Mechanical Universe and Beyond …"[11] was a public television series intended to serve as a calculus-based course in introductory physics. David Goodstein, a physics professor from the California Institute of Technology "plays" the professor, with real-life situations and

historical reenactments portrayed by actors, as well as impressive (especially so for the time period, more than 20 years ago) computer graphics. There are three-dimensional computer graphics scenes depicting electric force, field, and potential. These are notable for their simplicity and clarity. The series can be purchased on DVD or viewed online. Perhaps too dated to broadcast entire episodes to the classroom, excerpts are useful or teachers may wish to view the episodes for their own background.

Conclusion

Teaching high school students to visualize electric fields is a challenge. The electric field is invisible, intangible, and seemingly mysterious, but it is rooted in concrete experiences with charged objects. The foundations of the electric field concept are introduced when students begin vectors and the Newtonian force concept. Teachers who do a good job developing those topics will find the job of teaching electric field easier. Introducing the concept of field early on through gravitation and reinforcing the ideas with analogies to mechanics are tried-and-true techniques. A mixture of hands-on and virtual experiences can help students connect the abstract concept of lines of force with "the real world."

2004 AP Physics C Electricity and Magnetism Multiple-Choice Exam



Particles of charge Q and $-4Q$ are located on the x-axis as shown in the figure above. Assume the particles are isolated from all other charges.

45. Which of the following describes the direction of the electric field at point P?

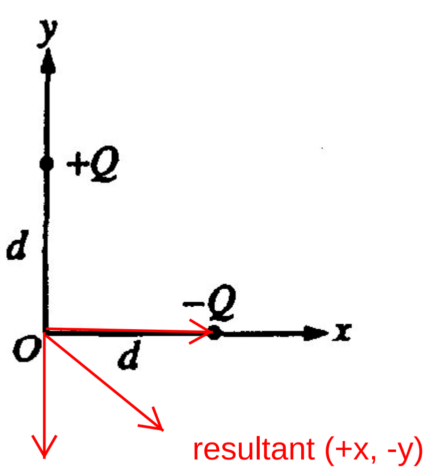
    (A) +x (B) +y (C) −y

    (D) Components in both the −x-and +y-directions

    (E) Components in both the +x- and −y-directions

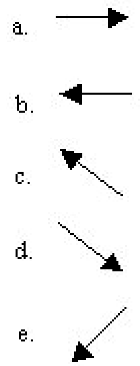
Answer: E. Electric field lines begin on positive charge and end on negative charge. The electric field from charge Q will point upward and to the right at P. The electric field from charge −4Q will point downward and to the right at P. Both charges are the same distance from P and both produce a field which points to the right. Since the magnitude of the −4Q charge is greater, its field in the y dimension cancels the field from the +Q charge and the y-component of the field at point P is downward.

2004 Physics B Exam

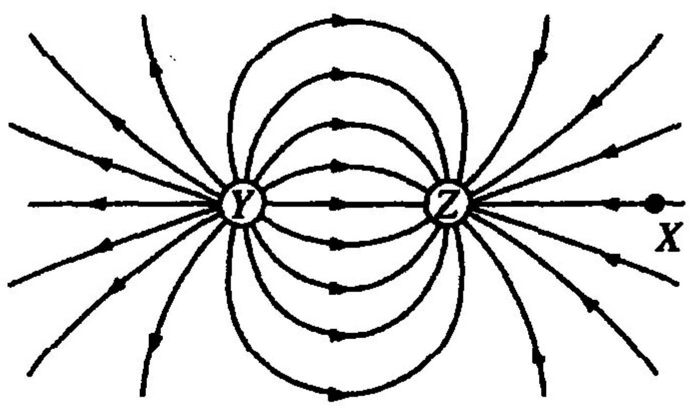


Charges $-Q$ and $+Q$ are located on the x- and y-axes, respectively, each at a distance d from the origin O, as shown above.

19. What is the direction of the electric field at the origin *O*?



Answer: D. The field from the positive charge at upper left points downward at O and the field from the negative charge at lower right points right at O. The net field is therefore directed diagonally down and to the right.



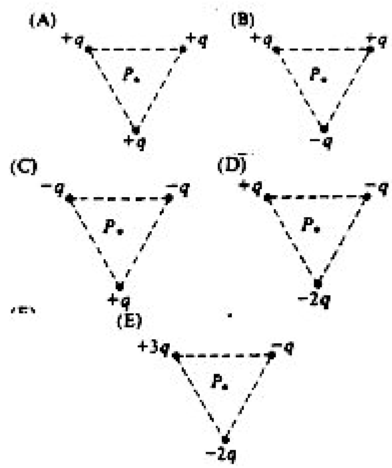
66. The diagram above shows electric field lines in an isolated region of space containing two small charged spheres, *Y* and *Z*. Which of the following statements is true?
    - a. The charge on *Y* is negative and the charge on *Z* is positive.
    - b. The strength of the electric field is the same everywhere.
    - c. The electric field is strongest midway between *Y* and *Z*.
    - d. A small negatively charged object placed at point *X* would tend to move toward the right.
    - e. Both charged spheres *Y* and *Z* carry charge of the same sign.

Answer: D. Since the arrows point away from sphere Y and toward sphere Z, Y is positive and Z is negative. Since both spheres have the same number of field lines, their charges are equal in magnitude. A positively charged object feels a force in the direction of the field lines, but the

object is negatively charged, and feels forces opposite to the field lines. Sphere Y would attract and sphere Z would repel the small object. Since electric force is proportional to $1/R^2$ and the object is closer to sphere Z, the repulsion is stronger.

AP Physics C 1988 Multiple-Choice Questions—Electricity and Magnetism

Questions 47–48 relate to the following configurations of electric charges located at the vertices of an equilateral triangle. Point P is equidistant from the charges.



47. In which configuration is the electric field at P equal to zero?

(A) (B) (C) (D) (E)

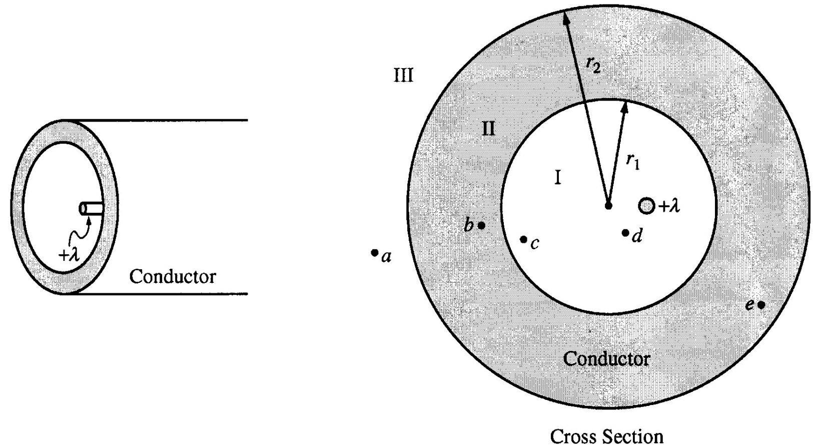
Answer: A. All configurations have charges equidistant from Point P. Therefore, only those configurations with equal magnitudes can cancel the field, since it is proportional to q/R^2. Choices A, B, and C all have equal magnitude of charges, but only A cancels the field. All three field vectors are equal in length. Arrange them in a tip-tail vector diagram. Since the diagram ends where it started, the resultant is zero, and the field cancels.

48. In which configuration is the electric field at P pointed at the midpoint between two of the charges?

(A) (B) (C) (D) (E)

Answer: C. The x-components of the field from the two upper charges cancel. The field from the lower charge points straight up to the top of the page. The y-components of the two upper charges also point to the top of the page. The net field is pointed to the midpoint of the uppermost side of the triangle.

2004 Physics C Exam



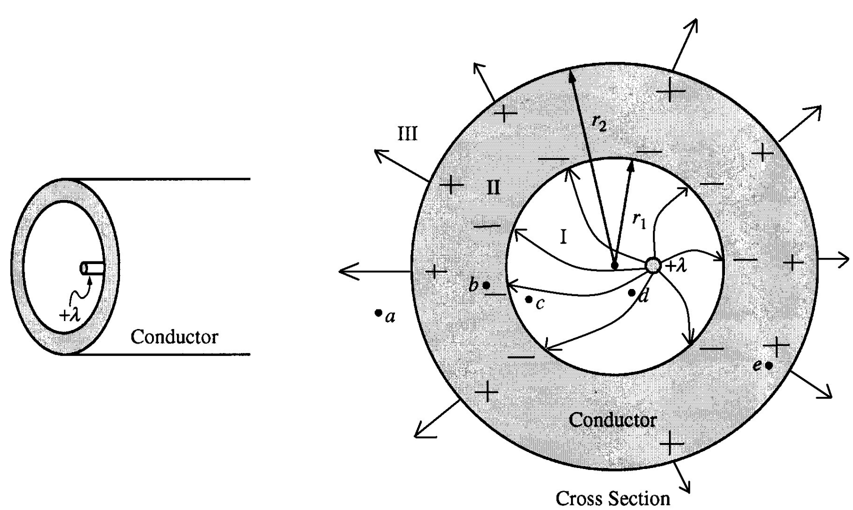
2004E1 (part a) The figure above left shows a hollow, infinite, cylindrical, uncharged conducting shell of inner radius r_1 and outer radius r_2. An infinite line charge of linear charge density $+\lambda$ is parallel to its axis but off center. An enlarged cross section of the cylindrical shell is shown above right.

a. On the cross section above right,

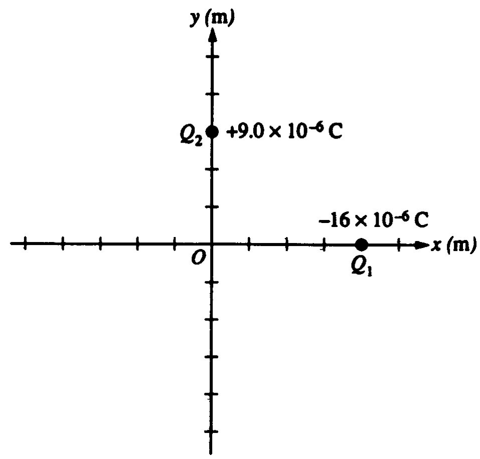
   i. sketch the electric field lines, if any, in each of regions I, II, and III and

   ii. use + and − signs to indicate any charge induced on the conductor.

Solution: The infinite line charge induces a negative charge in the adjacent surface of the shell. A corresponding positive charge is induced on the outer surface of the shell. The field is stronger on the right side where the line charge is closer to the shell, therefore the field lines are closer together. The field lines leave the shell radially and the line charge almost radially (it is difficult to draw radial lines given the space provided).



1993 Physics B Exam



1993B2. A charge $Q_1 = -16 \times 10^{-6}$ coulomb is fixed on the x-axis at $+4.0$ meters, and a charge

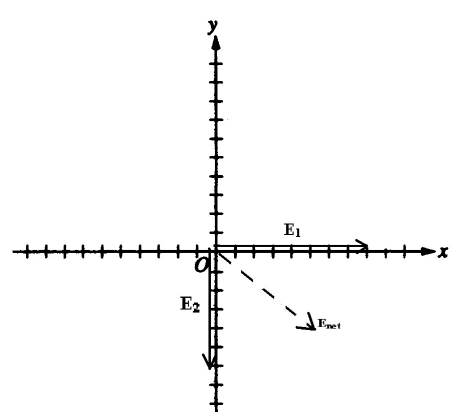
$Q_2 = +9 \times 10^{-6}$ coulomb is fixed on the y-axis at $+3.0$ meters, as shown on the diagram above.

a. i. Calculate the magnitude of the electric field E_1 at the origin O due to charge Q_1.
ii. Calculate the magnitude of the electric field E_2 at the origin O due to charge Q_2.
iii. On the axes below, draw and label vectors to show the electric fields E_1 and E_2 due to each charge, and also indicate the resultant electric field E at the origin.

Solution: i. $\mathrm{E} = k\frac{q}{r^2} = 9 \times 10^9 \frac{N \cdot m^2}{C^2} \times \frac{16 \times 10^{-6}\, C}{(4m)^2} = 9000 N/C$

ii. $\mathrm{E} = k\frac{q}{r^2} = 9 \times 10^9 \frac{N \cdot m^2}{C^2} \times \frac{9 \times 10^{-6}\, C}{(3m)^2} = 9000 N/C$

iii. The field from the positive charge will point straight down at the origin, the field from the negative charge will point straight to the right at the origin. The net field is down and to the right at negative 45 degrees.



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1. Modeling Curriculum: Second Year Materials (available to modeling workshop attendees). Contains experiments, demonstrations, lesson plans, worksheets, quizzes, and tests for teaching electrostatics. To find a list of currently scheduled workshops, visit http://modeling.asu.edu.

2. PTRA Resource Material: Teaching About Electrostatics, by Robert A. Morse. Available from American Association of Physics Teachers (http://www.aapt.org).

3. The PhET Web site contains (among many others) a simulation of what happens when a balloon is rubbed on a sweater. http://phet.colorado.edu/web-pages/simulations-base.html.

4. Newton, Sir Isaac, *Principia* (*On the Shoulders of Giants*), p. 428, Philadelphia, Penn.: Running Press Publishers, 2005.

5. Hecht, Eugene *Physics: Calculus*, pp. 624–625, Pacific Grove, Ca.: Brooks/Cole, Second edition, 2000.

6. University of Arkansas, Fayetteville, University Physics II Web site, http://www.uark.edu/depts/physinfo/up2/guide/courseguide.htm, "Chapter 7, Electric Field Maps."

7. O'Kuma, Thomas L.; Maloney, David P.; Hieggelke, Curtis J. *Ranking Task Exercises in Physics*, Upper Saddle River, N.J.: Prentice Hall Inc., 2000.

8. A list of some notable Web sites offering tools to visualize the electric field:

    a. MIT Open CourseWare, Electricity and Magnetism, Spring 2005. http://ocw.mit.edu/OcwWeb/Physics/8-02TSpring-2005/Visualizations/index.htm
    b. PhET Charges and Fields http://phet.colorado.edu/simulations/chargesandfields/ChargesAndFields.swf
    c. Physlet Simulations and Animations for Second-Semester Physics http://physics.bu.edu/~duffy/semester2/semester2.html
    d. Electric Field http://www.nep.chubu.ac.jp/~kamikawa/electricfield/elefi_e.htm
    e. Presenting . . . Your Electric Field! http://qbx6.ltu.edu/s_schneider/physlets/main/efield.shtml
    f. Electrostatic Fields and Potentials http://www.physics.brocku.ca/applets/Coulomb/
    g. Electric Fields and the Force on a Charge http://www.mhhe.com/physsci/physical/giambattista/electric/electric_fields.html
    h. Paul Falstad's Math and Physics Applets http://www.falstad.com/mathphysics.html

9. EM Field 6 and Electric Field Hockey are available from Physics Academic Software (http://webassign.net/pas/em_field/emf.html; http://webassign.net/pas/electric_field_hockey/efh.html).

10. "Visualizing Potential Surfaces with a Spreadsheet." Robert J. Beichner, *The Physics Teacher*, Vol. 35, pp. 95–97, February 1997.

11. "The Mechanical Universe and Beyond …" a public television series produced in 1985 is available for purchase or to be viewed over the Web at: http://www.learner.org/resources/series42.html.

Electric Potential and Potential Energy

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The concepts of electric potential and potential energy are often confusing to students, since the word "potential" is used so often with various shades of meaning. We tend to use the terms "potential," "potential energy," "potential difference," and "voltage" too loosely for beginning students to develop a solid grasp of the meanings. Extra time and emphasis spent on these terms will have huge payoffs for students as they move on to other more advanced topics in electrostatics and electrodynamics.

In the lesson presentations below, it is recommended that teachers present the terms, equations, and diagrams visually—writing them on the board or presenting them on a screen as they are first mentioned. To follow each lesson, sample assignments are presented, along with other recommended resources and activities to help students visualize these rather abstract concepts. Teacher presentation notes are also included in italics within the text.

The topic "Electricity and Magnetism" constitutes an entire semester for Physics C, so much more in-depth study is necessary, including the use of calculus in solutions. The first lessons presented here provide the basic presentation of topics for Physics B—and a beginning for what is needed in Physics C.

Lesson 1: Introduction to Electric Potential Energy

Review of Electric Charge

Electric charge, represented by the symbols Q or q, is a property of matter. The smallest charge found isolated in nature is the charge on one electron (we'll write it e^-), which is $\mathbf{-1.6 \times 10^{-19}}$ **Coulombs**. "Like" charges (positive-positive or negative-negative) exert forces of repulsion on each other, and "unlike" charges (positive-negative) exert forces of attraction on each other.

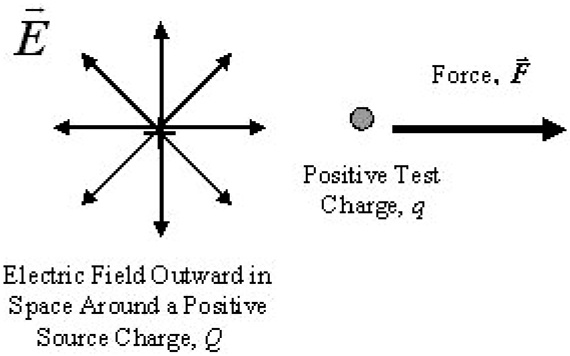
Review of Electric Field

Electric field, E, describes a region in space in which a small positive charge, called a **test charge,** q**,** will experience an **electric force**, **F**. Electric field vectors are directed outward in all directions around a positive **source charge**, Q, from the surface of the charge to infinity. Electric field vectors are directed inward toward a negative charge from infinity to the surface of the charge.

The magnitude of the electrical force between two charges, where r is the separation between the two charges, is:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{r^2} \text{ (Coulomb's Law)}$$

A representation of the electric field around a positive point charge appears below. As you can see, the force on a test charge placed in the field is in the direction of the field line in the vicinity of the test charge.



For <u>positive</u> charges in electric fields, the electric field vectors and force vectors are in the same direction. For negative charges in electric fields, the electric field and force vectors are in the opposite direction. This is made obvious by the vector equation $\boldsymbol{F} = q\boldsymbol{E}$, where $\boldsymbol{F}$ and $\boldsymbol{E}$ are in the same direction when q is positive and in the opposite direction when q is negative.

[Note to teacher: Illustrate this in the classroom by drawing the positive source charges on the board in red and negative source charges in blue—labeling each with + and − signs. Of course, remind students that electrical charges aren't really "red" or "blue"! Then use a small red object—such as the cap from a marker—to carry in your hand to represent the small positive test charge. Move the "test charge" to different positions around each source charge, emphasizing and exaggerating the force of each field on the test charge in your hand. Include movements in and out of the board near the source charge to emphasize the three-dimensional nature of the electric field around each source charge.]

Review of Electric Flux

Electric flux, Φ_E, describes the "flow" of electric field lines through an area perpendicular to the direction the lines are pointing: $\Phi_E = EA$. The units of flux are $\mathrm{N \cdot m^2/C}$. Electric flux remains constant, regardless of distance from a charge. For example, if the distance from a charge doubles, the surface area of the imaginary sphere defined by that radius is four times as great (from the mathematical formula for surface area of a sphere: $A = 4\pi r^2$). However, as the distance from a charge is doubled, the electric field at that distance is 1/4 as great ($E = kQ/r^2$), so flux remains constant. Ultimately, the flux depends only on the magnitude of the source charge. [Do not confuse electric flux, Φ_E, with magnetic flux Φ_B, which is measured in webers and will be studied later. The concept of electric flux will be more important in the Physics C course, where Gauss's Law is applied.]

Electric Potential Energy

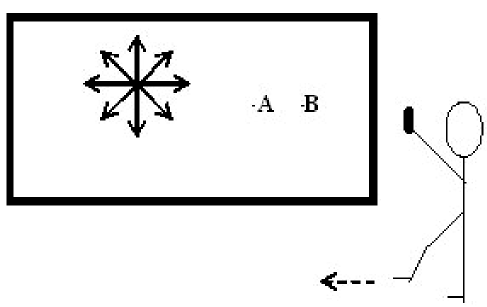
Let's suppose we start with the test charge at an infinite distance from our source charge. [*The source charge is still in red on the board. Position yourself as far from the board as possible in*

the room, with the red pen cap in hand. Students will enjoy imagining that you have traveled to infinity!] Now, as you move the positive test charge toward the positive source charge, it will require positive work to do so, since there is an electric force trying to push the test charge away. Thus, you have to do positive work to bring the test charge closer and place it at point A (see diagram below) in the field near the source charge. [*Walk to the board with the red "test charge" and mark point A on the board.*] Since positive work was done in moving the test charge to this position, the test charge must have **electric potential energy** when placed at point A.

Can you imagine what would happen if we released the test charge at point A? Yes, it would begin to move back to infinity, as the potential energy of the charge at point A is converted to kinetic energy. **Conservation of energy** applies to a charge in an electric field. The total energy of the test charge must remain constant, as potential energy is converted to kinetic energy when the charge is released.

Now go back to infinity with the test charge and repeat the process, this time moving it to a position a little farther from the source charge. Label it point B. Since the test charge was not brought as close to the source charge as before, less work was required, so the charge has less potential energy at point B. Moving the test charge from point B to point A would require positive work, because we would be <u>increasing</u> the potential energy of the charge. Moving the test charge from point A to point B would require negative work, since we would be <u>decreasing</u> the potential energy of the charge.

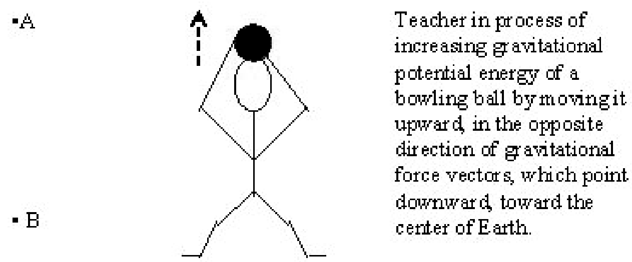
> Electric potential energy, U_E, is measured in **joules**, and work done in moving a charge in an electric field is also measured in joules.



Teacher in process of increasing electric potential energy of positive test charge (i.e., red cap) by moving toward the positive source charge, in the opposite direction of electric field lines.

We have experience with objects, such as things with mass, that move spontaneously from higher potential energy positions to lower potential energy positions when released in a gravitational field. An object, such as a bowling ball, requires a force against gravity to lift it toward the ceiling. As it is moved against this force, the ball's gravitational potential energy

increases. [*Ask students what is happening to the gravitational potential energy of the bowling ball as you lift it, then ask them what they think would happen if you released it!*] Similarly, a positive test charge has an increased electrical potential energy as it is moved against the electrical force to position it near a positive source charge. The positive test charge would move spontaneously from A to B when released (as in the diagram above), since it has higher potential energy at A than it has at B—in the same way the bowling ball would move spontaneously if released at a point above the teacher's head.



Lesson 1 Activities

1. *Visualizing Charges*: Use brightly colored red and blue objects to represent positive and negative charges, respectively. Hold the objects representing test charges up in your hand as you "walk" to different positions around source charges to provide a visual for students to "see" the effects of forces and fields. (The cap of a whiteboard marker works well.) The more you "play it up," by feigning a "push" or "pull" on the charge in your hand, the better students will be able to visualize these concepts.

2. *Electric Field Model*: Stick thin drink straws into a Styrofoam ball, spacing them as uniformly as possible all over the ball, to represent electric field lines. Students can then see the three-dimensional aspect of the field—and can also see that the number of field lines (straws) remains constant, getting closer together near the ball and farther apart as you move away from the ball. Point out that: (1) Electric field vectors are tangent to the field lines. (2) Electric field strength is greater where the field lines are closer together and weaker where the lines are farther apart. (3) Field lines never cross. (4) The **flux**, or total number of lines, does not change as you get farther from the charge (sphere). (5) The **flux density**, or number of lines through a given area perpendicular to the lines decreases farther from the charge (sphere). (6) Though the number of lines drawn or shown in a model is finite—depending upon the size of the charge—electric field actually exists at every point in space. This model is simply a representation of the three—dimensional properties of the field.

3. *Electric Flux Model*: (Class demonstration or group activity) Use a thick-walled round balloon, such as available at party supply stores. Cut out a square about 2 cm by 2 cm in the middle of an index card. On the deflated balloon, make dots all over the surface, spaced about 1/2 cm from each other. Blow up the balloon so it is just lightly inflated and hold or clamp the valve closed (do not tie). Measure the radius, then hold the card to the balloon's

surface and count the number of dots visible. (Average over several areas.) The dots represent electric field lines passing through the surface of the balloon from an imaginary positive charge located at the center of the balloon. Now inflate the balloon so it is about double in size and calculate the new surface area. Again, use the card on the surface to get an average of number of dots visible. (a) What is the general relationship between surface area and number of dots (electric field lines moving through the area on the card)? *Ans: These should be in inverse proportion. Students could be asked to graph the relationship.* (b) What happens to the density of "dots" (field lines) as the balloon gets larger (i.e., distance from the charge at the center increases)? *Ans: Density should decrease, which would represent a weaker electric field.* However, it's important to stress that field lines are just representations of fields, which are continuous at every point around charges. (c) What happens to the total number of "dots" on the balloon as it is inflated? *Ans: The number of "dots" on the balloon (electric field lines) stays constant, representing the constant flux from the charge— an indication of the size of the source charge, which does not change.*

4. *Videotape*: The *Mechanical Universe* program, "Electric Fields and Forces" has excellent graphics that portray the three-dimensional nature of electric fields. Forces among collections of charges are also shown, along with the concept of flux.

5. *Field Physlets*: The Davidson College Web site has short Physlets that describe electric fields around a single charge, multiple charges, and lines of charge. These could be shown during class and manipulated by the teacher, or students could be given exercises on the Web site as homework. An example might be: "Play the 'line of charge' Physlet under the category Electric Fields and sketch, using vectors, the field strength around a line of positive charge."

Lesson 1 Sample Assessment Questions (*with Answers*)

1. Two equal positive charges are placed on the x-axis at $x = 2$ and at $x = 4$.

   (a) Where on the x-axis would the electric field due to those two charges be equal to zero?

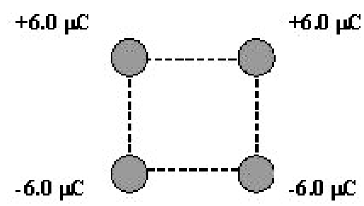
   Answer: At $x = 3$, the net electric field would be zero, since the electric field vectors from the two charges would be equal in magnitude and opposite in direction at that point.

   (b) What would be the net electric force due to the two positive charges on an electron placed at $x = 3$?

   Answer: Since $\mathbf{F} = q\mathbf{E}$, there would be no force on any third charge placed at $x = 3$, where $\mathbf{E} = 0$.

Lesson 1 Sample Assessment Problems

2. Four 6.0 μC charges are held in position in a square configuration as shown below. The length of each side of the square is 4.0 μm, and the magnitude of each charge is 6.0 μC.



(a) Calculate the magnitude and direction of the electric field at the center of the square.

Solution: First, draw the electric field vectors at the center due to each of the charges. (Toward the center for the positives and away from the center for the negatives.)

Then consider the x-components and y-components of each vector separately. These components will all be equal, since we have a square, and each component is equal to the magnitude of the field vector from one charge times cosine 45°. Next, we want to look for symmetry, i.e., situations where vectors are equal and opposite and will add to zero. The x-components of the two $+6\mu C$ charges meet "head on"—and will cancel—one to the right and one to the left. The same thing will happen with the x-components of the two $-6\mu C$ charges. All the x-components have now cancelled to 0, so we only need to consider the y-components. Electric field is outward toward the center from the positives, so their y-components will be down on the page. Electric field is inward from the center toward the negatives; these y-components are also down on the page from the center of the square.

$$E_x = 0$$

$$E_y = 4\left(\frac{k(6 \times 10^{-6}\ C)}{(2\sqrt{2} \times 10^{-6})^2}\right) \cos 45° = 1.9 \times 10^{16}\ \text{N/C downward on the page}$$

(b) Determine the magnitude and direction of the electric force on an electron placed at the center of the square.

Solution: $\boldsymbol{F} = q\boldsymbol{E} = (-1.6 \times 10^{-19}\ C)(1.9 \times 10^{16}\ \text{N/C downward}) = 3.0 \times 10^{-3}\ \text{N}$ upward

(c) Calculate the magnitude and direction of the initial acceleration of the electron at the moment it is released—but the other four charges are held in position.

Solution: $\boldsymbol{a} = \boldsymbol{F}/m = (3.0 \times 10^{-3}\ \text{N upward})/(9.1 \times 10^{-31}\ \text{kg}) = 3.3 \times 10^{27}\ \text{m/s}^2$ upward

[*Note: The acceleration has this value only initially. Since the force varies with distance from the other charges, the acceleration will also change in value as the electron changes position after its initial acceleration.*]

(d) How will the electric field at the center of the square change if the two charges on the right side of the square are exchanged with each other?

*Solution: Draw the electric field vectors at the center due to each of the charges to show that they are equal and opposite—i.e., they are all of equal magnitude with two toward the center and two away from the center, so they cancel and **E** = 0.*

Lesson 2: Electric Potential Energy and Introduction to Electric Potentials, Equipotentials, and Potential Difference

Electric Potential, Potential Difference, and Equipotentials

Electric potential, or **absolute potential,** V**,** is defined as the potential energy per unit charge. It can be considered as the amount of potential energy a certain charge gains when it is moved from infinity to a position in an electric field. The potential energy would also be equal to the amount of work required to move the charge to this given position. Since absolute potential is defined in terms of energy, work, and charge, it is a scalar quantity.

$$V = \frac{U_E}{q} \quad \text{or} \quad V = \frac{W}{q}$$

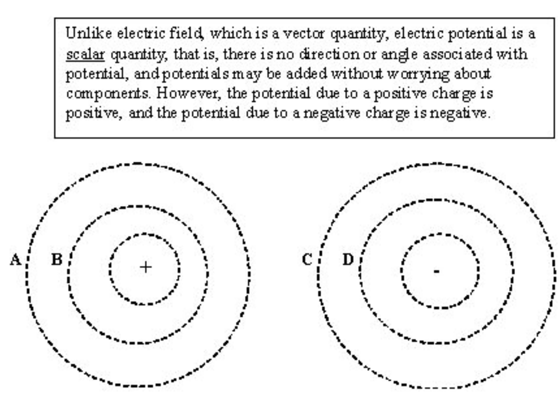
Absolute potential, which is sometimes just called electric potential, V, is measured in **volts.**

Since $V = \frac{U_E}{q}$: **1 volt = 1 joule/coulomb**

Electric potential can also be defined in terms of a source charge, Q, that produces an electric field:

$$V = \frac{U_E}{q} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q}{r}$$

Let's examine what the above equation means: The electric potential, V**,** around a positive source charge, Q, has a higher positive value near the charge and lower positive value farther from the charge. As the distance, r, from the charge increases, the potential, V, decreases. You can also see from the above equation that if the source charge, Q, is negative, the potential around it is also negative. As the distance, r, from the charge increases, the potential around the charge is less negative.

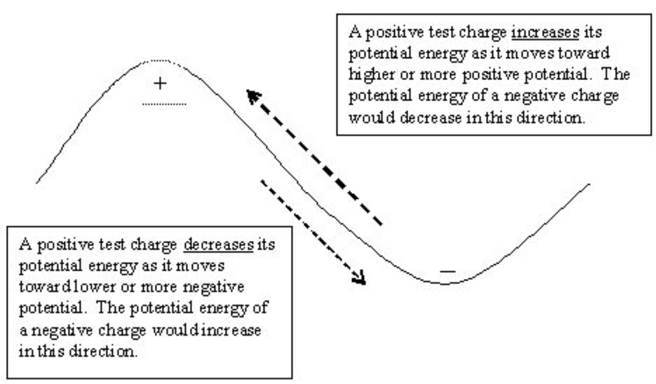


In the diagram above, the electric potential at point B is positive—and greater than the positive potential at point A**.** We can say that there is a **potential difference, ΔV,** between those two points. For example, if the potential at point A is 20 volts and the potential at point B is 30 volts, then the potential difference from A to B, $\Delta V = V_B - V_A$, is 10 volts. The ΔV from B to A is -10 volts. A charge moving from point A to point B would experience an *increase* in absolute potential, or a *positive gradient* in potential. Likewise, since the potential values at points C and D are negative-and the potential at C has a smaller negative value than the potential at D— then $V_C > V_D$. For example, if the potential at point D is -30 volts and the potential at point C is -20 volts, then point C has a higher potential value than the potential at point D. The potential difference, ΔV, from point C to point D is:

$$\Delta V = V_D - V_C = (-30 \text{ v}) - (-20 \text{ v}) = -10 \text{ v}$$

A positive test charge would move spontaneously from point C to point D, since that would be moving from higher potential to lower potential—like a mass rolling downhill! This is confirmed by the negative value for ΔV from point C to point D. [Note: The ΔV from point D to point C is $+10$ volts, so a positive test charge would not move spontaneously from D to C.]

The electric potential energy of a charge can be compared to gravitational potential energy. A positive source charge can be compared to the "top of a hill," where electric potential is positive. A positive gradient in potential is like "going up the hill." Electric potential gets less positive farther from the positive source charge—or farther "downhill." Likewise, a negative source charge can be thought of as the "bottom of a valley." This concept can be compared to moving a mass (which is always positive) up a hill, where its gravitational potential energy is larger. Likewise, moving a positive charge away from the positive source charge (or "downhill") will decrease its potential energy.



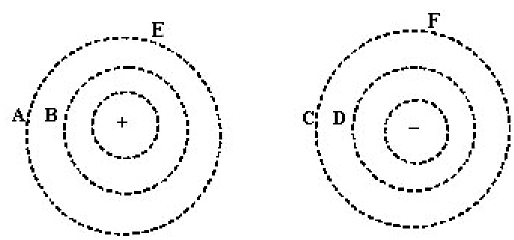
Now, let's consider what happens to a negative test charge moving toward a negative source charge—this time using the equation, $\Delta U = q\Delta V$. If the test charge q is negative and the source charge is negative, then both Δq and ΔV are negative. The negative charge, then, increases its potential energy as it is moved closer to the negative source charge. [Since we don't experience "negative masses" this is difficult to picture in our gravitational analogy.

But in a world with "negative masses," those masses would have higher potential energy in a valley than on a hill—just the opposite, right?]

In the situation we have described, *positive charges would naturally move from higher potential to lower potential*—just as an object would roll downhill. Negative charges would naturally move from lower potential to higher potential—just as "negative masses" would roll from valleys to the tops of hills in an imaginary world of "negative masses." This movement of charges due to potential differences will be very important in the discussion of how current moves in circuits.

Positive charges move spontaneously from higher potential to lower potential, and negative charges move from lower potential to higher potential.

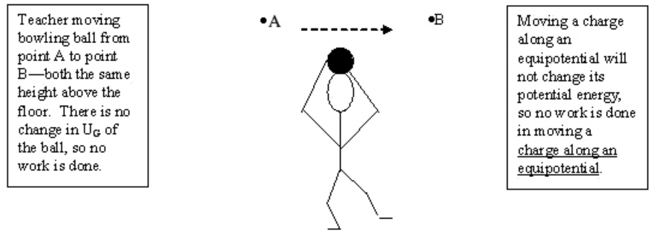
Now, let's take another look at the potentials around the positive and negative test charges. *Remember that those potentials exist in three dimensions—in all directions around each charge.*



Any point in space around a charge that is at the same distance**,** *r*, from the charge will have the same **absolute potential** (sometimes just called potential). Imaginary "surfaces" that connect these points are called **equipotentials**. The potential*, V*, has the same value at every point along an equipotential surface surrounding a charge. Some equipotentials (in only two dimensions) are drawn around the positive and negative charges above. In the diagram, the points A and E lie along an equipotential, so these points are at the same potential. Likewise, the negative potential at C is equal to the negative potential at F. Thus, there is no potential difference between these points**:** $V_A = V_E$ and $V_C = V_F$. We can say there is a **potential difference,** ΔV**,** between points A and B and between points C and D**.** [This potential difference is sometimes just called **voltage—**and will be the basis for movement of charges in the discussion of circuits later on.] The amount of work done in moving any charge in an electric field is due only to the potential difference that exists between the two points:

$$W = q\,\Delta V$$

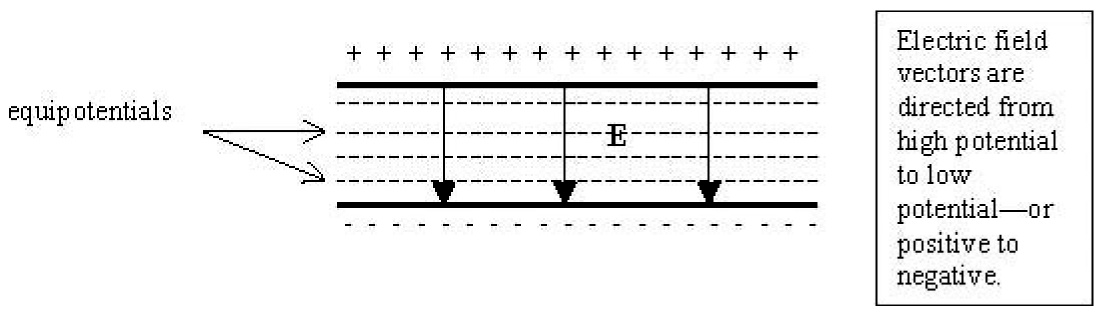
Moving a charge along an equipotential is much like moving a bowling ball between two points in the room that are the same height above the floor. There is no change in gravitational potential energy, so no work is done.



[*Note to teacher: It's important here to model all this by drawing the source charges with potential lines on a board and using those red and blue "pencap" charges to show what might happen in various situations—positive test charge near both positive and negative source charges, and negative test charge near both positive and negative source charges.*]

Remember, electric field is the *force per unit charge* and is a vector, and electric potential is the *work or potential energy per unit charge* and is a <u>scalar</u>.

We can create a uniform electric field by charging two parallel metal plates held in position. The upper plate has a positive net charge, and the lower plate has an equal amount of negative net charge. The dashed parallel lines represent equipotentials, or all points that are the same distance from the upper plate or from the lower plate.



Since the electric field is uniform, it's helpful to define the electric field strength in terms of the potential difference between the charged plates and the distance between the plates:

$$E = \frac{\Delta V}{d}$$

In this equation, the derived units for electric field will be volts per meter (V/m). Previously, we had derived units for electric field of newtons per coulomb (N/C). It would not be difficult to prove that the two sets of units are equivalent (i.e., 1 V/m = 1 N/C), so either is an appropriate unit for electric field.

Charged metal conducting plates which are connected as shown previously are called **capacitors**—devices that store charge on the plates and store electrical energy in the electric

field. The amount of stored charge depends upon the size of the capacitor and the potential difference between the plates:

$$Q = C\Delta V$$

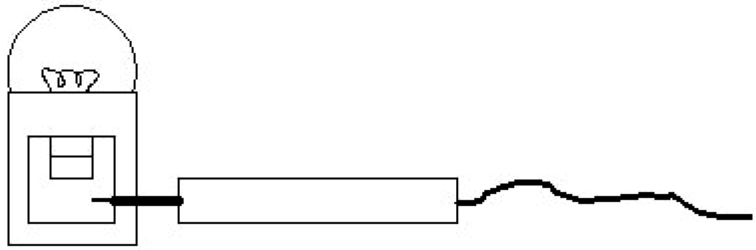
The amount of energy stored in a capacitor depends upon the same factors:

$$U = \tfrac{1}{2}\, Q\Delta V = \tfrac{1}{2}\, C(\Delta V)^2$$

Capacitance is measured in **Farads**, where 1 F = 1 $\frac{C}{V}$

Lesson 2 Activities

1. *Van de Graaff Generator*: Select a student with fine, medium-length hair and have the student stand on a plastic stool near the Van de Graaff generator. Instruct the student to place his or her fingers lightly on the globe of the discharged, unplugged generator. Both the generator and student are at zero potential, with no potential difference between them. As the generator is turned on and charged, the student gains charge so that both are at equal potential. After the peremptory "hair standing on end," have the student release his or her fingers but remain on the plastic stand as the generator is turned off. The student should retain the charge until he or she steps off onto the floor—at which point the hair will instantly fall as the student is "grounded," i.e., equalizes charge with the Earth. [*Make sure the student has no metal attachments* (*belt buckles, shoe buckles, piercings*) *that could arc with nearby metal parts on cabinets, etc. It's important to model safety with the generator.*]
2. *Tesla Coil Plasma Bulb*: Before you begin, tell students that if they see a spark in air today that is about 2–3 cm long, there must be a potential difference of about 50,000 volts. [The break-down voltage of air—or potential difference that will cause the gases in air to ionize and allow a current to pass—is 50,000 volts per inch.] Set a large clear globe-type light bulb on top of the box in which it was purchased, so that the base of the bulb is visible through the open part of the box. Darken the room and turn a handheld Tesla coil up to about 50,000 volts, holding the tip an inch or so from the base of the bulb. Students should see a spark jump between the Tesla coil and light bulb base. Now have a trusted assistant hold the coil and touch the base of the bulb. The teacher can then lightly touch the top of the glass globe, causing sparks to jump from the filament to the point where fingers are touching—because the teacher is at very low potential. Students should also note the difference in color of sparks inside and outside the bulb. [*Note: Though the Tesla coil produces the high potential with alternating current, the concept of potential difference is nicely demonstrated here—and you don't have to mention the alternating current. I also don't like to allow students to touch the coil or the bulb, since this "plasma bulb" demonstration is not as safe as the commercial versions.*]



3. *Videotapes*: *The Mechanical Universe* programs "Potential Difference and Capacitance" and "Equipotentials and Fields"

Lesson 2 Sample Assessment Questions Fill-in-the Blank (*with Answers*)

1. (*Negative*) work is done by an external force in moving an electron closer to a stationary positive charge. (*Positive*) work is done by an external force in moving a proton closer to a stationary positive charge. (*No*) work is done in moving an electron along an equipotential line.
2. The electric potential energy of a positive test charge will (*increase*) as it is moved farther from a negative source charge. The electric potential energy of a negative charge will (*increase*) as it is moved closer to another negative charge.
3. As the distance from a charge doubles, the strength of the electric field due to that charge is (*decreased*) by a factor of (*one-fourth)*, and the magnitude of the electric potential is (*decreased*) by a factor of (*one-half*).
4. Electric field is measured in the units (*N/C or V/m*). Electric potential is measured in (*volts*), electric potential difference is measured in (*volts*), and electric potential energy is measured in (*joules*).
5. A 200 μF capacitor is charged to a potential difference of 10 volts. Compare the charge stored in the same capacitor if it is charged to 20 volts.

   Answer: Since Q = CV, doubling the potential difference on the capacitor would also double the charge stored in the capacitor.

Lesson 2 Sample Assessment Problems (*with Solutions*)

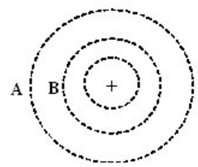
1. Calculate the potential energy of a proton when it is located 3.0×10^{-4} m from a 6.0 μm charge. Also calculate the work done by an external force in moving a proton to that point and the work that would be done by the electric field to move the proton to the same point.

   Solution: $U_E = qV = (q)\left(\frac{kQ}{r}\right) = \frac{(1.6 \times 10^{-19}\ C)(9 \times 10^{9}\ Nm^2/C^2)(6 \times 10^{-6}\ C)}{3 \times 10^{-4} m} = 2.9 \times 10^{-11} J$

   The work required by an external force to position the proton at a given location is equal to the potential energy of the proton at that location, or 2.9×10^{-11} J. The work done on the proton by the electric field would be negative 2.9×10^{-11} J. (Think about it: The field

would be exerting a force pushing the proton away—as the proton is moved closer. Thus, the force due to the field is in the opposite direction of the direction of motion, and the work is negative.)

2. In the situation here, a positive 2.0 μC charge is held in position. Point **B** is 1.2×10^{-4} m from the charge, and point **A** is 1.8×10^{-4} m from the charge.



(a) Calculate the absolute potential at point **B** due to the positive charge.

Solution: $V_B = kq/r = \dfrac{(9 \times 10^9\ Nm^2/C^2)(2 \times 10^{-6}\ C)}{1.2 \times 10^{-4}\ m} = 1.5 \times 10^8\ V$

(b) Calculate the potential difference from point **B** to point **A.**

Solution: $V_A = kq/r = \dfrac{(9 \times 10^9\ Nm^2/C^2)(2 \times 10^{-6}\ C)}{1.8 \times 10^{-4}\ m} = 1.0 \times 10^8\ V$

$$\Delta V = V_f - V_0 = V_A - V_B = 1.0 \times 10^8 - 1.5 \times 10^8 = -5.0 \times 10^7\ V$$

(c) Determine the work done by the electric field in moving a proton from **B** to **A.**

Solution: $\Delta U_E = q\Delta V = q(V_f - V_0) = (+1.6 \times 10^{-19}\ C)(1.0 \times 10^8 V - 1.5 \times 10^8 V)$

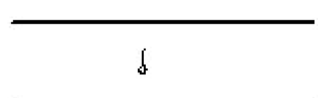
$$\Delta U_E = -8.0 \times 10^{-12}\ J$$

The potential energy change of the proton is negative, so the work done by the electric field is positive, since it is conservative. Another way to think of this is that the electric force on the proton and the displacement of the proton in the electric field are vectors that are in the same direction, so the work done by the field is positive.

$$W = +8.0 \times 10^{-12}\ J$$

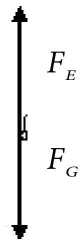
(d) Determine the work done by an external force in moving a proton from **B** to **A.** *Solution: The magnitude of the work is the same as in part* (*c*)*, but the work is negative, since the potential energy of the proton is being decreased.* (*This would be analogous to lowering a bowling ball from the ceiling to the floor. The potential energy is decreased, so you do negative work on the ball as you lower it to the floor.*)

3. The well-known Millikan oil drop experiment involved measurements of negatively charged oil droplets as they moved or were suspended between two charged horizontal parallel plates. Assume in the situation below that the droplet has a mass of 2.0 mg and the potential difference between the two charged plates is 10 volts. The distance between the plates is 0.012 m. At the time of observation, the droplet is suspended (so we can neglect any viscous drag on the droplet).



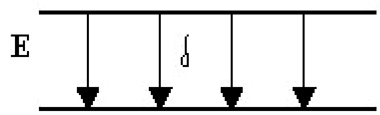
(a) Draw a free body diagram showing the forces on the droplet.

Solution:



Since the droplet is not moving, the forces are in equilibrium, with gravitational force balanced by the electrical force.

(b) Draw arrows between the two plates showing the direction of the electric field between the plates.



*By **F**= q**E**, since the charge on the droplet is negative, the force and field are in opposite directions. The electric field in this case must be downward if the electrical force is upward.*

(c) Calculate the magnitude of the electric charge on the oil droplet.

Solution: $\boldsymbol{F_G} = \boldsymbol{F_E}$

$$mg = qE = q\Delta V/d$$

$$(2.0 \times 10^{-6}\ kg)(9.8\ m/s^2) = \frac{q(10\ V)}{0.012\ m}$$

$$q = 2.4 \times 10^{-8}\ C$$

(d) Determine approximately how many excess electrons were acquired by the droplet during charging.

Solution: Since the charge on each electron is 1.6×10^{-19} *C, the number of electrons is the total charge divided by charge per electron.*

$$\# = (2.4 \times 10^{-8}\ C)/(1.6 \times 10^{-19}\ C) = 1.4 \times 10^{12}\ electrons$$

Lesson 3: Potentials, Potential Energy, and Work for a Collection of Charges

Electric potential is a scalar quantity, so adding potentials for a collection of point charges is rather simple. To find the potential at a given point, simply find the value of the potential at that point due to each of the source charges—and add.

$$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$$

To find the potential energy of a single charge located among a group of charges, first find the potential at the location of the single charge due to the other charges-using the method shown above. Then multiply the charge at that location by the potential at that location to find the potential energy of the charge:

$$U_E = qV = (q) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

The potential energy of a group of charges can be found using the following steps:

(1) Find the electric potential energy between any two charges, using the formula

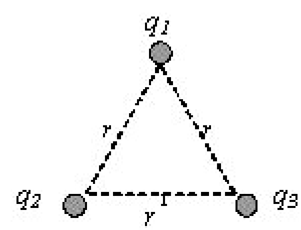
$$U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

(2) Repeat the calculation between each pair of charges.

(3) Add the electric potential energies for all the pairs of charges to find the total potential energy of the group of charges.

The work required to assemble a group of charges is equal to the potential energy of the collection of charges.

Let's suppose we have three charges located equidistant from each other (below):



The total potential energy of this collection of charges is the sum of the potential energies of the pairs of charges:

$$U_E = k\left[\frac{q_1 q_2}{r} + \frac{q_1 q_3}{r} + \frac{q_2 q_3}{r}\right]$$

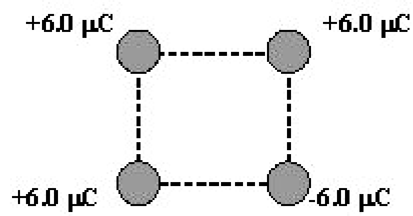
Lesson 3 Sample Assessment Questions

1. Which would require a larger amount of work to assemble in the triangular arrangement shown above—3 equal positive charges, 3 equal negative charges, or two positive and one negative charge?

Answer: The amount of work to assemble them would be equal to the sum of the potential energy of each pair. For the three positive charges, we would be adding three positive potential energies. For the three negative charges, we would be adding the same three quantities—since the product of the two negative charges in each term would be positive. The amount of work in both cases would be the same. For the two positive and one negative charges, we would be adding one positive term and two negative terms—all of equal magnitude-so the net work would be less.

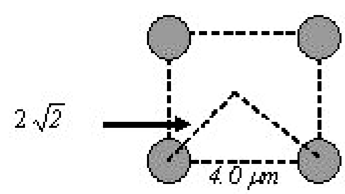
Lesson 3 Sample Assessment Problems:

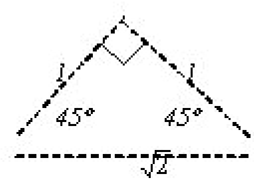
1. Four charges are held in position in a square configuration as shown below. The length of each side of the square is 4.0 μm, and the magnitude of each charge is 6.0 μC.



(**a**) Calculate the absolute potential at the center of the square.

Solution: First, it's important to use trigonometric relationships when examining configurations of charges. Then we need to note any "symmetries" in the arrangement. To calculate the potential at the center, we need the distance of each charge from the center (same for all). On closer examination, we see sets of 1-1-$\sqrt{2}$ right triangles, with "legs" equal to the distance needed.



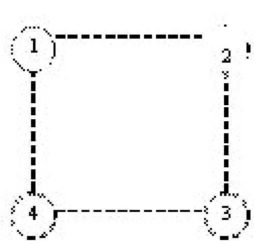


Since one of the positive potentials will cancel one of the negative potentials, the potential, a scalar quantity, is the sum of the potentials from two of the positive charges.

$$V_{net} = \sum V_i = 2\left[\frac{k(6 \times 10^{-6}\ C)}{2\sqrt{2} \times 10^{-6}}\right] = 3.8 \times 10^{10}\ V$$

(b) Determine the potential energy of the system of four charges.

Solution: The potential energy of the system is the sum of the potential energies of the pairs. There are six possible combinations. (To help clarify this, we'll label the charges 1-2-3-4, starting with the charge on the upper left and moving clockwise.)



$$U_{12} = \frac{kq_1q_2}{r} = \frac{k(+6\mu C)\,(+6\mu C)}{(4\mu m)}$$

$$U_{13} = \frac{kq_1q_3}{r} = \frac{k(+6\mu C)\,(-6\mu C)}{4\sqrt{2}\,\mu m}$$

$$U_{14} = \frac{kq_1q_4}{r} = \frac{k(+6\mu C)\,(+6\mu C)}{4\mu m}$$

$$U_{23} = \frac{kq_1q_2}{r} = \frac{k(+6\mu C)\,(-6\mu C)}{4\mu m}$$

$$U_{24} = \frac{kq_1q_2}{r} = \frac{k(+6\mu C)\,(+6\mu C)}{4\sqrt{2}\,\mu m}$$

$$U_{34} = \frac{kq_3q_4}{r} = \frac{k(+6\mu C)\,(-6\mu C)}{4\mu m}$$

$$\boxed{\begin{array}{c} U_{12} + U_{34} = 0 \\ \textit{and} \\ U_{13} + U_{24} = 0 \\ \textit{and} \\ U_{14} + U_{23} = 0 \\ \text{So: } \sum U = 0 \end{array}}$$

(c) Calculate the potential energy of a 10 μC charge placed at the center of the square.

Solution: $U_E = qV = (10 \times 10^{-6}\ C)(3.8 \times 10^{10}\ V) = 3.8 \times 10^{5}\ J$

(d) Determine the work required to move the 10 μC from infinity to the center of the square.

Solution: The net work required to move the charge to that position would be the same as the potential energy it has at that position. $W = 3.8 \times 10^{5}\ J$

(e) What would be the electric potential at the center of the square if the charge on the upper left corner is negative instead of positive?

Solution: In this situation, the sum of the two positive and two negative values for potential—all of equal magnitude and the same distance from the center—would be zero. Remember that electric potentials are scalar quantities that are simple. Additionally, if the potential at the center is zero, any charge—regardless of its size or amount of charge—would have no potential energy at that point. (It's also important to note that the electric potential at the center of the square would be zero regardless of where the two positive charges and two negative charges are placed, as long as they're all the same distance form the center.)

2. A set of charged parallel conducting metal plates are separated by a distance of 0.04 cm and have a potential difference between them of 6 volts. An electron is released from a position near the negative plate. Determine the speed of the electron as it hits the positive plate.

Answer: First, calculate the electric potential energy of the electron as it is released.

$$U_E = q\Delta V = (1.6 \times 10^{-19}\ C)(6\ V) = 9.6 \times 10^{-19}\ J$$

In the absence of any nonconservative forces acting on the electron, the total energy of the electron remains constant. When the electron meets the positive plate, all the potential energy has been converted to kinetic energy.

$$K = \tfrac{1}{2}mv^2 = 9.6 \times 10^{-19}\ J$$
or

$$\tfrac{1}{2}mv^2 = q\Delta V$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{(2)(1.6 \times 10^{-19}\ C)(6\ V)}{(9.1 \times 10^{-31}\ \text{kg})}} = 1.5 \times 10^6\ m/s$$

Lesson 4: Extended Topics for AP Physics C

The first three lessons cover topics that are applicable to both Physics B and Physics C courses. Electricity and Magnetism topics comprise only 25 *percent of a yearlong course in AP Physics B. The Physics C course, however, includes much more extensive coverage, including the use of calculus in solving problems, since it is designed to be an entire semester course. The resources below can help provide more extensive coverage of the topics listed here. The Rensselaer Polytechnic Institute Web site* (*cited below*) *has very helpful concept review and interactive practice problems that are specific to Physics C.*

Vector Notation

Vector properties of electric fields and electric forces require a more concise statement of the equations than provided on the Physics B equation sheet. Since force and field are both vectors, the equations must be consistent; i.e., showing vector expressions on both sides of each equation:

$$\boldsymbol{F} = k\frac{Qq}{r^2}\,\hat{\boldsymbol{r}}$$

The $\hat{\boldsymbol{r}}$ (read: "r-hat") notation is a unit vector that simply means "radially outward"—giving the right side a vector direction but not changing the magnitude of the calculation. Now both sides of each equation are vectors. In the force equation, the force has the calculated magnitude, with the force directed radially from the charge Q to the charge q. If both charges have the same sign, the force from Q to q is *outward*, causing repulsion of the charges. If only one of the two charges is negative, the force is $-\hat{\boldsymbol{r}}$, the opposite of outward, which is *inward*, so the force is an attractive force.

$$\boldsymbol{E} = k\frac{Q}{r^2}\,\hat{\boldsymbol{r}}$$

In this electric field equation, it is now clear that the electric field is radially *outward* if the source charge, Q, is positive. If the source charge is negative, the direction of the electric field is $-\hat{\boldsymbol{r}}$, or radially *inward*.

Gauss's Law

Gauss's Law is an important tool in solving problems in AP Physics C. The law states that any closed surface, such as a sphere, that encloses a charge, has a net flux through the closed surface that is proportional to the amount of charge enclosed:

$$\Phi_E = \oint \boldsymbol{E} \cdot d\boldsymbol{A} = \frac{q}{\varepsilon_0}$$

Gauss's Law is treated in depth in a separate focus article.

Electric Field, Potential, and Potential Difference with Calculus

When a positive charge, q, is moved between two points A and B in an electric field, the change in potential energy can be calculated using the formula:

$$\Delta U = -q \int_A^B \boldsymbol{E} \cdot d\boldsymbol{s}$$

The potential difference between the points A and B in the field is:

$$\Delta V = \frac{\Delta V}{q} = -\int_A^B \boldsymbol{E} \cdot d\boldsymbol{s}$$

In a uniform electric field, the above equations are often simplified to the form:

$$\Delta V = -Ed$$

where E is electric field strength, d is distance traveled along electric field lines, and the negative sign indicates that the direction of increasing potential is opposite the direction of the electric field vectors. For example, electric potential will increase as one moves nearer a positive charge—in the opposite direction of electric field.

The meaning is made more precise by use of the gradient form $\mathbf{E} = -\vec{\nabla} V$, which states that the electric field vector is in the opposite direction of the gradient of the potential. [You might think of electric field vectors pointing "downhill" from a positive charge and the gradient of V going "uphill" … thus they are in the opposite direction … and the negative sign.] This is consistent with our earlier discussion of potential, in which we showed that a positive charge is the source of electric field vectors that are directed radially outward—but the potential around that charge increases closer to the source charge. [*Note: Read the above equation "E equals negative grad V." The* $\vec{\nabla}$ notation is "grad" or gradient, or *d/dr.*] Looking back at the charged capacitor plates discussed in Lesson 2, where the top plate was positively charged and the bottom plate negatively charged, the gradient of the electric potential (lower to higher) would be toward the top plate. Using $\mathbf{E} = -\vec{\nabla} V$, the electric field is in the opposite direction, or downward in the diagram (from the top plate to the bottom plate).

Recommended Resources

(Interactive Web sites)
http://www.physics.sa.umich.edu/demolab/em.asp [B and C]
http://webphysics.davidson.edu/physlet_resources/bu_semester2/index.html [B and C]
http://webphysics.davidson.edu/physlet_resources/bu_physlab/index.html [B and C]
http://links.math.rpi.edu/webhtml/EMindex.html [Physics C, with calculus]

(Videos on tape or DVD)
The Mechanical Universe: [B and C] http://www.learner.org/series42.html
Program 17—"Electric Fields and Forces"
Program 18—"Potential Difference and Capacitance"
Program 19—"Equipotentials and Fields"

R. Beichner, "Visualizing Potential Surfaces with a Spreadsheet," *The Physics Teacher*, Vol. 35, pp. 95–97, February 1997. (Article with directions for creating three-dimensional graphs of electric potential.)

References

Halliday, David, Robert Resnick, and Jearl Walker. *Fundamentals of Physics*. New YorK: Wiley, 2001. [C]
Serway,Raymond A., Robert J. Beichner, and John W. Jewett, Jr. *Physics for Scientists and Engineers*. Fort Worth:Saunders College Publishing, 2000. [C]
Walker, James, *Physics, 3rd edition* [B]

Acknowledgment

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Teaching about Gauss's Law

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Gauss's Law is a key concept in any calculus-based introductory physics course. It is used to calculate the dependence of the electric field on the distance from charged conductors and insulators. It is also used to determine the location of excess charges on cylindrical and spherical conductors. And it is one of the most difficult concepts for students to fully understand because it relates to abstract concepts such as electric field and flux.

Students have everyday experiences with the concepts of mechanics such as force, velocity, acceleration, energy, etc. As Randall Knight[1] points out, they still harbor many misconceptions about these ideas, but at least they know that their teacher is trying to predict and describe motion using these concepts.

With electricity and magnetism, however, they have very few day-to-day experiences that help them understand the concepts of flux and electric field. Their only experience with static electricity may be when they pull clothes out of the dryer or get a spark when touching a doorknob.

Development of the Concepts

Prior to a discussion of Gauss's Law, students should understand the nature of forces between charged particles. They should have solved problems related to forces between charged objects (in particular, point charges), and they should be acquainted with the concept of an electric field as the force per unit charge on a positive test charge. It is often useful to the students for the instructor to draw parallels between the electric field of a point charge and the gravitational field of the earth. In fact, throughout the study of electrostatics, the instructor should explicitly show the students the parallels between electric field and gravitational field, between electric potential energy and between gravitational potential energy, etc.

Once the students understand the concept of electric field, they should be introduced to the concept of electric flux. Lillian McDermott et al. have written a series of tutorials[2] that develop this particular concept particularly well. The tutorial entitled "Electric Field and Flux" uses graph paper to develop the concept of an area vector, **A**, and a small area vector element, **dA**, for each of the squares on the graph paper.

1. Knight, R. *Five Easy Lessons: Strategies for Successful Physics Teaching*. San Francisco: Addison Wesley, 2002.

2. McDermott, L.C., and P.S. Shaffer. *Tutorials In Introductory Physics*. Upper Saddle River, New Jersey: Prentice Hall, 2002.

The tutorial then goes on to lead the student through several calculations of the ratio $\frac{\mathbf{F}}{q_{test}}$ for various values of q_{test} placed near a conducting rod. Once the students have determined that this ratio is constant for various values of q_{test}, this ratio is defined as the electric field. The concept of electric field lines is then developed for the student. Even in the case where this is not the first exposure for the student to the notion of electric field, it is a very useful reinforcement of the concept.

Once the area vector and the electric field have been defined and demonstrated for the student in this tutorial, the concept of flux is defined. The tutorial here requires the use of a block of wood with nails uniformly spaced through it. The students use a wire loop to represent the boundary of an imaginary flat surface. The nails represent electric field lines. The students are asked to orient the loop so the maximum number of lines pass through the surface of the loop and then so the minimum number pass through the loop. At this point, flux is defined and students are asked to draw electric field vectors (**E**) and area vectors (**A**) such that the flux is positive, negative, and zero. The last activity helps to develop the specific angle relationship between **E**, **A** and flux (i.e.,$\Phi = \mathbf{E} \cdot \mathbf{A} = |\mathbf{E}||\mathbf{A}| \cos\theta$ where θ is the angle between **E** and **A**).

This tutorial is an excellent introduction to the concepts of flux and electric field. As McDermott points out in the introduction, it can be used either as a cooperative learning activity or as an interactive lecture activity. They are designed to be used in small class settings where students work in groups of three to four as the instructor circulates, answering students' questions.

The second tutorial leads the students through a series of questions, the answers to which lead them to an understanding of Gauss's Law. They examine the flux through various closed cylinders and compare the flux to the charge enclosed in the cylinder. Once Gauss's law is stated ("*The electric flux through any closed surface is proportional to the net charge enclosed.*"), the tutorial leads the students through an application of Gauss's law to large sheets of charge density per unit area, $+\sigma$.

These tutorials are very useful for introducing the concepts of electric field, electric flux, and Gauss's Law and they provide an excellent introduction to a Gauss's law unit. If the instructor chooses not to use the tutorials, the concepts of electric field and flux must be developed prior to a discussion of Gauss's Law.

Application of the Concepts

Once the students have been introduced to Gauss's Law ($\Phi_E = \frac{q_{in}}{\varepsilon_0}$ where ε_0 is the permittivity of free space) they then need to be taught how it is used to understand electric field magnitudes and charge densities. First, they need to know that Gauss's law is useful for calculating the magnitude of the electric field only in highly symmetric situations where the flux through the Gaussian surfaces boils down to a simple multiplication. The three common symmetries to which Gauss's law is frequently applied are spherical, cylindrical,

and planar. It is important to emphasize to the student that we cannot apply Gauss's law if we do not already know a lot about the shape of the electric field created by a certain charge configuration.

We choose a Gaussian surface based on two criteria:

- The electric field must be perpendicular to the Gaussian surface at each point, or parallel to it. In equation form, $\mathbf{E} \cdot \mathbf{dA} = |\mathbf{E}||\mathbf{dA}|$ or $\mathbf{E} \cdot \mathbf{dA} = 0$
- If the electric field is perpendicular to the surface, then the magnitude of the electric field must be constant over the entire surface.

For example, if we want to use Gauss's law to learn about the radial dependence of the electric field due to charged conducting spherical shell, we need to know that the electric field is directed radially away from the spherical shell. And we need to know that the magnitude of the electric field is the same for all points at a specified distance from the center of the shell. This leads us to choose a spherical Gaussian surface, concentric with the shell. For curved surfaces, **dA** points in a different direction at each point. The flux is calculated using $\Phi_E = \oint \mathbf{E} \cdot \mathbf{dA}$ where the circle on the integral sign means that we are calculating the surface integral over a closed surface. With a judicious choice of Gaussian surface, $\oint \mathbf{E} \cdot \mathbf{dA}$ reduces to $|\mathbf{E}||\mathbf{A}|$ where $|\mathbf{A}|$ is the magnitude of the surface area of the entire Gaussian surface. For a spherical Gaussian surface, the area is equal to $4\pi r^2$ where r is the radius of the Gaussian sphere. So the flux is equal to $\Phi = (E)(4\pi r^2)$.

We can use spherical Gaussian surfaces to study many different charge configurations:

- point charge,
- hollow spherical conductor of radius R,
- solid spherical conductor of radius R,
- solid spherical insulator of radius R and uniform charge density ρ,
- a solid sphere inside and concentric with a hollow sphere,

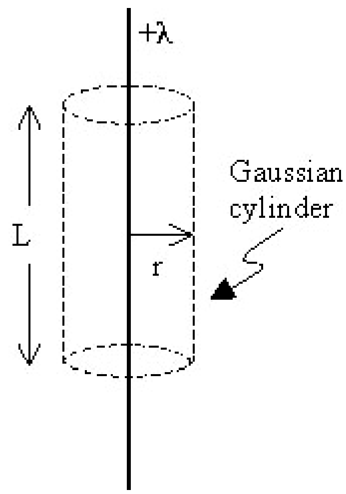
and many other combinations of these configurations. Most textbooks will use some or all of the above configurations in their example problems to demonstrate applications of Gauss's Law. It pays for the teacher to address all spherical charge distributions in one day before moving on to cylindrical distributions. It is also important to emphasize that the flux calculation will always end up being $\Phi = E \cdot 4\pi r^2$ for the spherical symmetry and the q_{in} can be the more difficult calculation, especially for insulators with non-uniform charge density.

There are lots of past AP problems that give examples of the kinds of calculations the students are expected to perform using Gauss's Law:

- calculate $E(r)$ inside the sphere,
- calculate $E(r)$ outside the sphere,

- graph $E(r)$ for $0 < r < \infty$,
- determine the location of excess charge on a conductor, and
- calculate the surface charge density on the inner and outer surfaces of a conductor,

just to name a few. Most of the Gauss's law questions on the AP Exam also involve calculations of the electric potential using $\Delta V = -\int \mathbf{E} \cdot \mathbf{ds}$, and even calculations of capacitance. Several past AP questions include a dielectric in part or all of a spherical (or cylindrical) capacitor. Thus, it is recommended that the instructor hold off on using these AP problems in class until after material on potential and capacitance has been taught. The following Gauss's Law problems from the Physics C: Electricity and Magnetism exam deal with spherical symmetry: 1979 Problem 1, 1981 Problem 1, 1983 Problem 1, 1989 Problem 1, 1990 Problem 1, 1992 Problem 1, 1996 Problem 1, 1999 Problem 1, 2003 Problem 1.



For cylindrical symmetries, it is easy to start by analyzing the field due to a long line of charge, of uniform density λ. By symmetry, one can argue that the field points radially away from the line and that all points a given distance from the line have the same magnitude of electric field. The students are generally quick to guess that a right circular cylinder, centered on the line of charge, is the appropriate Gaussian surface. The flux is then calculated over the three different surfaces: top, bottom, and sides (I often refer to that as the "soup label part")

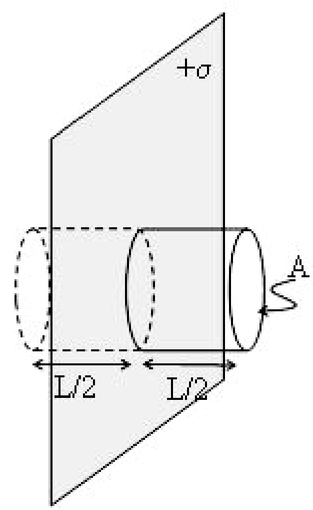
$$\oint \mathbf{E} \cdot \mathbf{dA} = \int_{top} \mathbf{E} \cdot \mathbf{dA} + \int_{sides} \mathbf{E} \cdot \mathbf{dA} + \int_{bottom} \mathbf{E} \cdot \mathbf{dA}$$

The flux through the top and bottom are zero since the field is parallel to those surfaces. The electric field is perpendicular to the sides, and constant in magnitude, so the flux is simply the product of the area and the magnitude of the field. Thus, the flux through the cylindrical Gaussian surface reduces to $\oint \mathbf{E} \cdot \mathbf{dA} = E \cdot A = E \cdot 2\pi r L$, where r is the radius and L is the length. The charge inside this Gaussian surface also depends on the length of the cylinder, so the L will drop out and the electric field will depend only on the charge density and the distance from the line.

Again, most calculus-based textbooks include several cylindrical charge distributions in their example problems. The instructor should discuss (or assign for homework) at least the following configurations:

- line of charge,
- cylindrical conducting shell,
- solid conducting cylinder.
- solid insulating cylinder with uniform charge density,

and concentric combinations of the above (i.e., a solid cylinder surrounded by a concentric cylindrical shell). The Physics C: Electricity and Magnetism exam has several Gauss problems with the cylindrical symmetry: 1985 Problem 1, 1993 Problem 1, 1995 Problem 1, 2000 Problem 3, and 2004 Problem 1.



The last symmetry to consider is planar symmetry. If the instructor used the aforementioned McDermott tutorials to develop the Gauss's Law concept, then it is probably sufficient to review by evaluating the field inside and outside a charged capacitor (i.e., two oppositely charged parallel plates of charge density σ). If not, it is best to start by analyzing a single sheet of surface charge density $+\sigma$. The field radiates away from the sheet on both sides like a bed of nails with the field lines perpendicular to the sheet. We don't need to know yet that the field is independent of distance from the sheet, only that it is the same magnitude at equal distances on either side of the sheet. We choose a Gaussian cylinder again, only this time, the axis is perpendicular to the sheet. We assume it is bisected by the plane so both ends are equidistant from the charge. The only flux is through the ends of the cylinder and thus, by a similar analysis to that done above is equal to $\Phi = \oint \mathbf{E} \cdot \mathbf{dA} = 2EA$ where A is the cross-sectional area of the cylinder. The charge enclosed is also proportional to A, and thus, it cancels out in the derivation of E. The electric field is thus found to be $E = \frac{\sigma}{\varepsilon_0}$. Planar symmetry is used on the Physics C: Electricity and Magnetism exam much less frequently.

See the following problems for examples: 1979 Problem 2, 1980 Problem 2, and 1984 Problem 2.

In summary, it is important to discuss all three symmetries for which Gauss's law is useful: spherical, cylindrical, and planar. It is helpful for the students to see several examples of each symmetry. Gauss's Law can be fairly easily taught in three or four days of discussion: one for spheres, one for cylinders, one for planar symmetry, and possibly one for review.

Problems to Practice

Mutliple-Choice Questions: Here is a sample of multiple-choice questions taken from previous AP Exams. More can be found on AP Central® and in College Board publications**.**

1998 Multiple-Choice Question 41. Gauss's law provides a convenient way to calculate the electric field outside and near each of the following isolated charged conductors EXCEPT a

A) large plate B) sphere C) cube
D) long, solid rod E) long, hollow cylinder

Answer: C—As discussed above, the field needs to be of constant magnitude at a specified distance from the charge distribution in order to easily apply Gauss's Law. The cube does not meet this criterion.

1993 Multiple-Choice Question 38. The net electric flux through a closed surface is

A) infinite only if there are no charges enclosed by the surface.
B) infinite only if the net charge enclosed by the surface is zero.
C) zero if only negative charges are enclosed by the surface.
D) zero if only positive charges are enclosed by the surface.
E) zero if the net charge enclosed by the surface is zero.

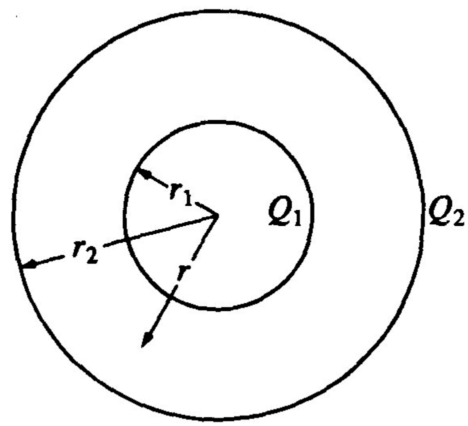
Answer: E—This is merely a statement of Gauss's Law

1993 Multiple-Choice Question 48. A conducting sphere of radius R carries a charge Q. Another conducting sphere has a radius $R/2$, but carries the same charge. The spheres are far apart. The ratio of the electric field near the surface of the smaller sphere to the field near the surface of the larger sphere is most nearly

A) 1/4 B) 1/2 C) 1 D) 2 E) 4

Answer: E—Gauss's Law shows that the field outside a conducting sphere is the same as if all the charge were concentrated at its center. Thus, the field is inversely proportional to the square of the radius, just as for a point charge.

1993 Multiple-Choice Questions 51–52. Two concentric, spherical conducting shells have radii r_1 and r_2 and charges Q_1 and Q_2, as shown. Let r be the distance from the center of the spheres and consider the region $r_1 < r < r_2$.



51. In this region the electric field is proportional to

A) Q_1/r^2 B) $(Q_1 + Q_2)/r^2$ C) $(Q_1 + Q_2)/r$
D) $Q_1/r_1 + Q_2/r$ E) $Q_1/r + Q_2/r_2$

Answer: A—According to Gauss's Law, the field at any point r is proportional only to the charge enclosed by a Gaussian surface with that radius.

52. In this region the electric potential relative to infinity is proportional to

A) Q_1/r^2 B) $(Q_1 + Q_2)/r^2$ C) $(Q_1 + Q_2)/r$
D) $Q_1/r_1 + Q_2/r$ E) $Q_1/r + Q_2/r_2$

Answer: E—When applying $V = -\int_{\infty}^{r} \mathbf{E} \cdot \mathbf{ds}$ to find the potential, the field $E_2 = \frac{k(Q_1 + Q_2)}{r^2}$ is used to calculate the field for $r > r_2$, and the field $E_1 = \frac{kQ_1}{r^2}$ is used for $r_1 < r < r_2$.

1993 Multiple-Choice Question 64. A solid nonconducting sphere of radius R has a charge Q uniformly distributed throughout its volume. A Gaussian surface of radius r with $r < R$ is used to calculate the magnitude of the electric field E at a distance r from the center of the sphere. Which of the following equations results from a correct application of Gauss's law for this situation?

A) $E(4\pi R^2) = Q/\varepsilon_0$ B) $E(4\pi r^2) = Q/\varepsilon_0$ C) $E(4\pi r^2) = (Q3r^3)/(\varepsilon_0 4\pi R)$
D) $E(4\pi r^2) = (Qr^3)/(\varepsilon_0 R^3)$ E) $E(4\pi r^2) = 0$

Answer: D—The charge inside is proportional to the volume enclosed by the Gaussian surface, which itself is proportional to r^3. Thus the student can eliminate all but choices C and D. The right side of the equation must have dimensions of Q/ε_0, thus leaving only choice D.

1988 Multiple-Choice Questions 56–57: Consider a sphere of radius R that has positive charge Q uniformly distributed on its surface.

56. Which of the following represents the magnitude of the electric field E and the potential V as functions of r, the distance from the center of the sphere, when $r < R$?

	E	V
(A)	0	kQ/R
(B)	0	kQ/r
(C)	0	0
(D)	kQ/r^2	0
(E)	kQ/R^2	0

Answer: A—The field inside is zero but the potential is not. The potential is the work per charge in bringing a charge in from infinity to R. Inside the field is zero so the potential is constant.

57. Which of the following represents the magnitude, of the electric field E and the potential V as functions of r, the distance from the center of sphere, when $r > R$?

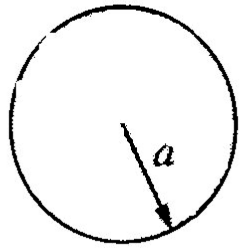
	E	V
(A)	kQ/R^2	kQ/R
(B)	kQ/R	kQ/R
(C)	kQ/R	kQ/r
(D)	kQ/r^2	kQ/r
(E)	kQ/r^2	kQ/r^2

Answer: D—According to Gauss's Law, these are the same as if the charge were concentrated at the center.

Dr. Chandralekha Singh and her colleagues at the University of Pittsburgh have developed a set of multiple-choice questions to survey introductory physics students' understanding of Gauss's Law.[3] They are excellent questions and valuable for probing student understanding of symmetry and superposition.

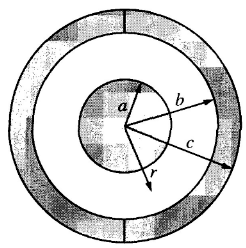
Free-Response Questions: Two examples of free-response problems from the AP Exam follow. Answers to these questions can be found in College Board publications, on the AP Central Web site, or at AP Summer Institutes and workshops.

3. Singh, C. "Student understanding of symmetry and Gauss's law of electricity." *American Journal of Physics*. October 2006.



1999 Physics C: Electricity & Magnetism Question 1. An isolated conducting sphere of radius $a =$ 0.20 m is at a potential of $-2{,}000$ V.

a. Determine the charge Q_0 on the sphere.

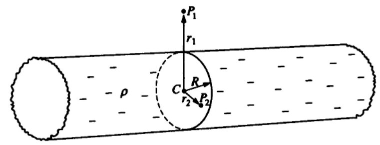


The charged sphere is then concentrically surrounded by two uncharged conducting hemispheres of inner radius $b = 0.40$ m and outer radius $c = 0.50$ m, which are joined together as shown above, forming a spherical capacitor. A wire is connected from the outer sphere to ground, and then removed.

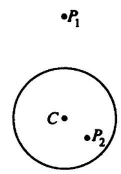
b. Determine the magnitude of the electric field in the following regions as a function of the distance r from the center of the inner sphere.
   i. $r < a$
   ii. $a < r < b$
   iii. $b < r < c$
   iv. $r > c$
c. Determine the magnitude of the potential difference between the sphere and the conducting shell.
d. Determine the capacitance of the spherical capacitor.

1993 Physics C Electricity** & **Magnetism Question 1, parts a and b.[4] The solid non-conducting cylinder of radius R shown above is very long. It contains a negative charge evenly distributed throughout the cylinder, with volume charge density ρ. Point P_1 is outside the cylinder at a distance r_1 from its center C and point P_2 is inside the cylinder at a distance r_2 from its center C. Both points are in the same plane, which is perpendicular to the axis of the cylinder.

4. Parts (c) and (d) pertain to Ampere's Law and are not relevant here.



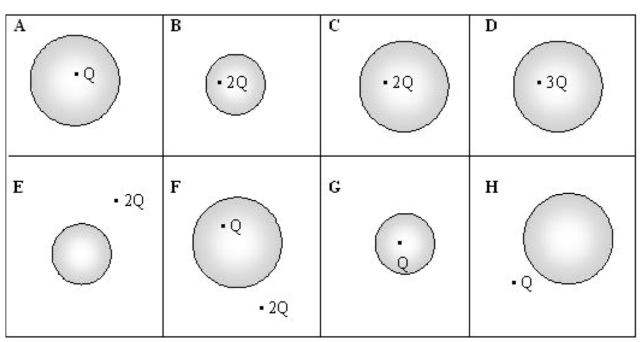
a. On the following cross-sectional diagram, draw vectors to indicate the directions of the electric field at points P_1 and P_2.



b. Using Gauss's law, derive expressions for the magnitude of the electric field E in terms of r, R, ρ and fundamental constants for the following two cases.
   i. $r > R$ (outside the cylinder)
   ii. $r < R$ (inside the cylinder)

Ranking Tasks: Ranking Tasks are a kind of physics problem inspired by physics education research. They have been developed mainly by David Maloney[5] and some of his colleagues.[6] In a ranking task, the student is presented with several contextually similar problems and asked to rank them based on one of the quantities relevant to this situation. The student is required to list them in order from greatest to least, and explain his or her reasoning. Several examples related to Gauss's Law follow.

***Ranking Task* #1**: Shown below are several spherical Gaussian surfaces, with positive charges present in or near them as shown. Rank the surfaces in terms of the net flux through the surface, with the sphere with the greatest net flux being first and the one with the least net flux being last. If two surfaces have the same amount of flux through them, give them the same ranking.



Greatest 1____2____3____4____5____6____7____8____ Least

5. Maloney, D. "Ranking Tasks: A New Type of Test Item." *Journal of College Science Teaching* (May 1987).

6. O'Kuma, T., Maloney, D. and Hieggelke, C. *Ranking Task Exercises in Physics.* Prentice Hall: Upper Saddle River, NJ., 2000.

Or, all of the spheres have the same net flux through them.________

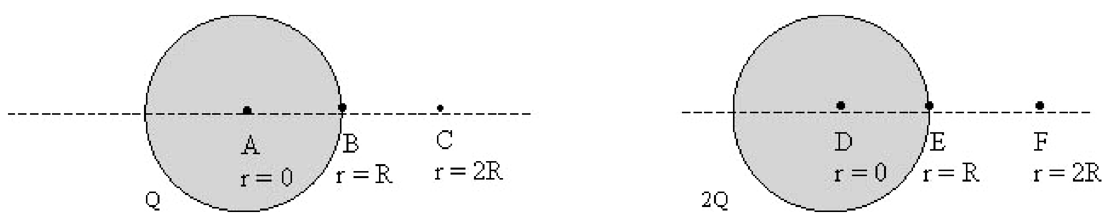
Please carefully explain your reasoning:

***Answer*:**
Greatest 1__**D**__2_**BC**__3_**AFG**__4_**EH**__ 5____6____7____8____ Least

***Explanantion*:** According to Gauss's Law, the flux is proportional to the charge inside the Gaussian surface. The outside charge is irrelevant.

***Ranking Task* #2**: Shown below are two clouds of charge, each of the same radius, *R*. The charge is uniformly distributed throughout each of the clouds. One cloud has a total charge *Q* and the other has a total charge 2*Q*.

Rank the labeled points from greatest to least in terms of the electric field magnitude at that location.



Greatest 1____2____3____4____5____6____ Least

Or, the electric field is the same at all points.________

Please carefully explain your reasoning.

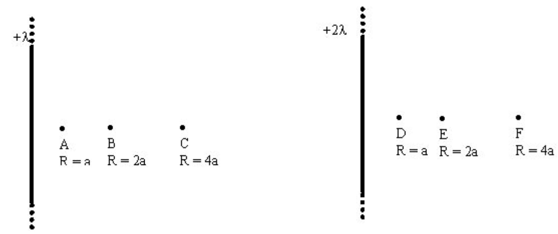
***Answer*:**

Greatest 1____**E**____2____**B**____3____**F**____4____**C**____5____**AD**____6____Least

Explanation: The field at the center is zero since a Gaussian surface of radius zero encloses no charge. So A and D are least. Then apply $E = kQ_{in}/r^2$ at each of the other points to obtain an algebraic expression for those.

***Ranking Task* #3**: Shown below are two infinite lines of charge, one of density $+\lambda$ and the other of density $+2\lambda$.

Rank the labeled points from greatest to least on the basis of the magnitude of the electric field at each of the points.



Greatest 1____2____3____4____5____6____ Least

Or, all the points have the same magnitude electric field.___________

Please carefully explain your reasoning:

Answer:

Greatest 1____**D**____2____**AE**____3____**BF**____4____**C**____5______6______Least

Explanation: According to Gauss's Law the magnitude of the electric field near a long wire is given by $E = \frac{\lambda}{2\pi\varepsilon_0 r}$. Simply plug in the relevant charge density and radius and find each magnitude.

Gauss's Law in the Laboratory

Labs related to Gauss's Law are few and far between. One reason for this may be that it is difficult to directly measure electric field. Lots of labs circumvent this problem by having the students measure electric potential using a simple voltmeter and then deduce the shape of the electric field from the equipotential lines. Two examples have been published in *The Physics Teacher* (a publication of the American Association of Physics Teachers) recently.

The first example involves the use of conducting paper and silver ink[7] (both available from PASCO.)[8] The student constructs a set of concentric circular conductors, the inner one is solid and has a radius of about 0.5 cm and the outer one a ring of radius about 7 cm (7–10 cm would fit on the paper). A potential difference of 10 V is then applied to the configuration. The students are then instructed to measure the potential (relative to ground at the outer ring) as a function of radius from the inner dot to the outer ring. The purpose of this lab is to determine if this electrode configuration simulates concentric spheres or concentric cylinders. The students must use Gauss's Law to calculate the electric field in between the conductors for both a spherical and a cylindrical configuration. They must then use $\Delta V = -\int \mathbf{E} \cdot \mathbf{ds}$ to determine the potential as a function of radius for each configuration.

7. Lietz, M. "A Potential Gauss's Law Lab." *The Physics Teacher* (April 2000).

8. Field Mapper Kit (PK-9023) from PASCO Scientific, 10101 Foothills Blvd., Roseville, CA 95678-9011 or http://www.pasco.com.

They then plug in specific numerical values for the voltage difference and radii to determine the numerical values in their theoretical derivations. The last step is to plot the data on the same axes with the theoretical predictions for both a cylinder and a sphere. The match between what a cylindrical theory predicts and the actual data is then very obvious to the student. In this lab, the data taking is very brief, but the calculations are a powerful review of two of the Gaussian symmetries.

Another example uses an "electric field probe" specifically designed for the purpose of studying Gauss's Law.[9] The probe has two conductors held 1.0 cm apart on a block of wood. Before knowing anything about electric potential, the students can be told that this probe measures the electric field in units of volts/centimeter. The students are then presented with an electrode configuration on conducting paper consisting of concentric rings (similar to the previous example). A constant potential difference is then applied. The students are then directed to find the magnitude and direction of the electric field. To find the direction, they "fix the location of the pin connected to the voltmeter ground, then 'walk' the other pin … in a circle around this point until the voltmeter reading is most negative."[10] This emphasizes the point that electric field points toward lower potential, just as a gravitational field points toward lower gravitational potential. They then move the probe along a radius and measure the field as a function of radius. The students are instructed to plot E vs. r and then E vs. $1/r$ to verify that this electrode configuration simulates concentric cylinders.

Both of these labs offer the students an excellent opportunity to see that Gauss's Law correctly predicts the electric field and potential as a function of radius for at least one electrode configuration. Electric fields will always be difficult for the students to visualize, but these two activities may help solidify their mathematical understanding of these concepts.

Conclusion

Hopefully these teaching techniques, from tutorials to ranking tasks, will help your students gain an appreciation for Gauss's Law. It is also helpful to make direct comparisons between Ampere's Law and Gauss's Law when magnetism is taught. Both are applicable to only very specific geometries, and both allow a calculation of the field based on charge (or current) enclosed in a surface (or encircled by a loop). Doing these comparisons will help solidify the understanding of both Ampere's Law and Gauss's Law.

9. Ludwingsen, D. and Hassold, G. "A Simple Electric Field Probe in a Gauss's Law Laboratory," *The Physics Teacher* (October 2006).

10. *Ibid*, 471.

Conceptual Links in Electrostatics

Using a Visual Mnemonic for Electrostatic Relationships

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One of the biggest challenges in teaching electrostatics is to help students realize that the concepts of force, field, potential energy, and potential are not disjoint ideas, but are linked to and derivable from each other. Even my best students have a hard time seeing these links, and are inclined to approach electrostatics from a purely mathematical perspective based on memorization of disjoint formulas. They can't ***visualize*** the problem as easily as they can in mechanics, so I frequently hear questions such as "What ***formula*** do I use?" or "How can I possibly ***memorize*** all these equations?" despite my best attempts to take a conceptual approach. All of us who teach electrostatics are very aware that developing a cadre of inquiry and hands-on activities such as we use in other areas of physics is a daunting if not impossible task. Randall D. Knight elaborates on the conceptual difficulty students have with electrostatics in his excellent book, which I highly recommend to anyone teaching physics.[11]

Rather than battling this tendency of the students to focus on electrostatic formulas, in recent years I decided instead to shamelessly exploit it. My primary goal may be the conceptual understanding of how all these concepts are linked together into a beautiful coherent framework, but the primary goal of most students is simply to be able to correctly work the problems, which they believe they can do if they just memorize those darned equations! I have observed many them making their own set of formula flash cards, not a practice I have recommended or encouraged. I finally realized that they do this because they feel a sense of confidence and control when they do memory work. After all, in previous science courses, biology in particular, they have found success through memorization. So, I developed a way of going along with this that I hope surreptitiously advances their understanding of the underlying physics concepts and how they are linked one to another.

Very close to the beginning of the electrostatic unit, I let the students know that there are a *lot* of equations. I introduce the concept map in Figure 1 as a visual mnemonic, and tell the students that Coulomb's Law is the only formula they will need to memorize provided they can learn how to properly use the map. This is admittedly a bit of an exaggeration, but it gets their attention. The formulas in the concept map work only for spherically symmetric charge distributions, and I emphasized this by placing them in circles. Using the concept map is easy. You memorize Coulomb's Law, scalar form. (The direction of the force is determined independently by looking at one particle or the other and considering the force as attractive or repulsive.) You then produce the other formulas from Coulomb's Law by simply "getting rid of a q" as you move to the right or "getting rid of an r" as you move down.

11. Knight, Randall D., *Five Easy Lessons.* San Francisco: Addison Wesley, 2002 191–235.

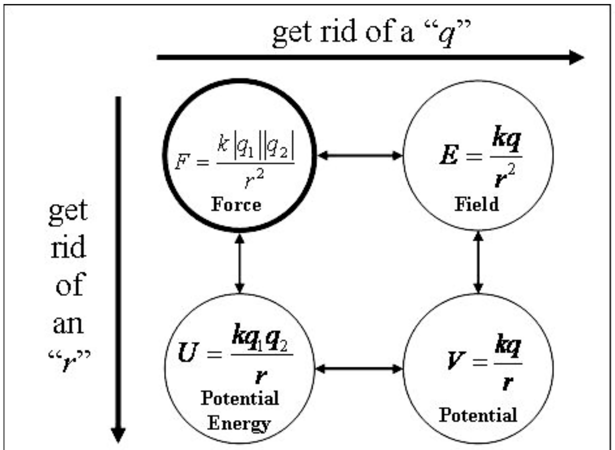


Figure 1: Concept Map

I fully realize I would be guilty of educational malpractice if this is where I stopped! But this concept map gives the students a level of comfort and a sense of control while, more importantly, providing a basis from which to investigate conceptually the links between forces, field, potential energy, and potential. So while the students are initially interested in what is inside the circles, I am most interested in the arrows that link the circles together.

The Link Between Force and Field

Electrostatic force, like all forces, arises from an interaction between two particles. Analysis of electrostatic force usually starts with a magnitude calculation using Coulomb's Law. Because only one magnitude calculation is performed, the student might believe that only one force exists rather than two equal and opposite forces, one on each particle. After a magnitude calculation using Coulomb's Law, the teacher might ask "what is the direction of the force?" The astute student will realize that this question cannot be answered unless the particle is first identified.

Figure 2 illustrates two charged bodies, q_1 and q_2, interacting to produce two forces, one on each charged body. Given a figure such as this one displaying only the electrostatic forces, the teacher might ask the student to characterize as fully as possible the signs and magnitudes of the charges. While it is obvious that the bodies are repelling each other, students are often unable to state that the charges must have the same sign but could be either positive or negative. Many of them will think that q_1 and q_2 must be of identical magnitude since the force vectors are the same length. Many students will also think that the charges must be accelerating in opposite directions; not a bad assumption if they remember Newton's Laws! The teacher should point out that in electrostatic systems, we must assume the presence of other forces that hold the charges in place, or otherwise they would indeed accelerate. Likewise, we assume in the rest of this instructional unit that the charges are not moving or accelerating unless otherwise indicated.

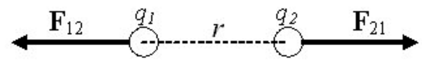


Figure 2: Electrostatic force requires two charges

Suppose we now make q_2 disappear, as is illustrated in Figure 3. The forces $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ disappear also, since they require two interacting bodies to exist. (However, these forces would certainly immediately reappear if q_2 were replaced.) We can say that the space around q_1 is modified by its very presence. In other words, there is an electric field around q_1 just waiting to create a force on q_2 or any other charged particle that happens to come along. The magnitude of the field around q_1 is calculated using an equation like Coulomb's Law but missing one of the charges, just like Figure 3 is missing one of the charges. (At this point, illustration of spherically symmetric electric fields would be appropriate, as would discussion of how the direction of the field is determined.)

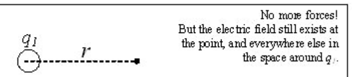


Figure 3: Electrostatic field requires one charge

Let's return to our concept map in Figure 1. When we want to obtain an equation for the electric field from Coulomb's Law, we move from left to right and get rid of a q. This is analogous to removing q_2 from Figure 2, which eliminates the forces but not the field due to the remaining charge q_1. When we move from right to left, from field to force, we reverse the process and replace the q. Hence

$$\mathbf{F} = q\mathbf{E}$$

The student should be informed that this "link equation" works always, and is not limited to situations of spherical symmetry.

The Link Between Potential Energy and Potential

Electrostatic scalars are even more abstract than their vector counterparts. The position of potential energy beneath force in Figure 1 will hopefully remind the student that electrostatic potential energy, U, also requires two interacting charges, q_1 and q_2. An important difference from forces is that only one potential energy exists for a given configuration of charges. If we make q_2 disappear from Figure 2, the potential energy disappears (but will reappear if q_2 is replaced). The space around q_1 is modified, as we saw in the section above. The electrostatic potential at a given point in this space predicts how much potential energy will be generated if a charge is moved into space surrounding q_1.

Potential energy requires two (or more) charges while potential requires just one. When we move from left to right on our concept map in Figure 1, from potential energy to potential, we get rid of a q. When we move from right to left, we replace the q. This enables us to calculate potential energy from potential.

$$U = qV$$

A more useful equation deals with the changes in potential energy and potential when a charge moves within the field generated by another charge (rather than appearing from infinitely far away).

$$\Delta U = q\Delta V$$

These equations work even when the symmetry is not spherical.

The Link Between Force and Potential Energy

Figure 1 shows how to obtain a formula for potential energy from Coulomb's Law by simply moving downward and "getting rid of an r." Looking at this simplistically, multiplication of Coulomb's Law by an r cancels an r in the denominator, thus eliminating the square. At this point, I will ask students if this sounds similar to anything they did in mechanics. It should, since the product of force and displacement is known as work.

$$W = \mathbf{F} \cdot \Delta\mathbf{r}$$

A visual representation of this appears in Figure 4.

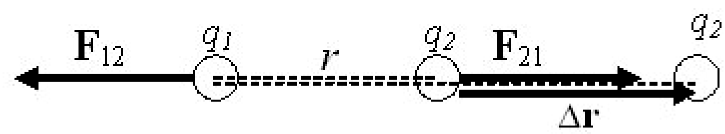


Figure 4: Work on q_2 due to force $\mathbf{F}_{21}$

Here, the force $\mathbf{F}_{21}$ acting on charge q_2 points in the direction of the displacement and does positive work on q_2. (The other force, $\mathbf{F}_{12}$, doesn't do work on q_2 since it does not act upon it.) This is, of course, a very oversimplified view of the situation, since the magnitude of $\mathbf{F}_{21}$ changes as r gets bigger. Physics C students should recognize this as an opportunity for an integral.

$$W = \int \mathbf{F} \cdot d\mathbf{r} = kq_1q_2 \int \frac{1}{r^2} dr$$

(The equation above assumes that the force and the displacement point in exactly the same direction.) Since the electrostatic force $\mathbf{F}_{21}$ is conservative, we are able to relate the work done by this force to a change in potential energy through the following relationship previously learned in mechanics.

$$-\Delta U = W = \mathbf{F} \cdot \Delta r$$

This means that the change in potential energy of a system can be related to the negative of the work done on it by the force. If the force pushes particle q_2 all the way to infinity, the potential energy will disappear entirely, leading to the following analysis.

$$\Delta U = U_{final} - U_{initial} = 0 - U_{initial} = -W$$

$$U_{initial} = W$$

So, the potential energy of the initial configuration is equal to the work the electric field will do as it moves q_2 from a distance r away from q_1 to an infinite distance away.

Going in the other direction, from potential energy to force, requires that we use the concept of the gradient of the potential energy, or the change in potential energy with respect to position. Problems of this nature appear in Physics C. From the expression for potential energy change above, we know that if the displacement is infinitesimally small

$$-dU = Fdr$$

and this can be rewritten as

$$F = -\frac{dU}{dr}$$

In a one dimensional system, this derivative is generally easily calculated. In multiple dimensions, derivatives must be taken independently for each dimension, or

$$F_x = -\frac{\partial U}{\partial x}; F_y = -\frac{\partial U}{\partial y}; F_y = -\frac{\partial U}{\partial y};$$

and the force, expressed as a vector, is

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

Graphical problems involving the determination of a force from potential energy will require that the student take a slope at a given point on a graph of potential energy as a function of position and equate the negative of this value to the force.

The Link between Field and Potential

Going from field to potential also requires "getting rid of an r" according to Figure 1. Since the r^2 term is in the denominator of the field equation, this requires a multiplication. The equation below is analogous to the one needed to calculate potential energy change from force and displacement.

$$\Delta V = -\mathbf{E} \cdot \Delta \mathbf{r}$$

This is pretty abstract stuff, one step removed from energy and forces. If the student has developed a somewhat better feel for energy by this point, then ask him to consider the following equation, which relates potential energy change to force and displacement. You might want to review the development of this equation again.

$$\Delta U = -\mathbf{F} \cdot \Delta \mathbf{r}$$

Now remove a q (i.e., divide by q) on both sides. Removing a q from potential energy, U, leaves us with potential, V, and removal of one from force, **F**, leaves us with field, **E**, remember?

In Physics B, most problems involve uniform fields, and we would not go any further; however, Physics C students might need to handle non-uniform fields with an integral such as the one below.

$$\Delta V = -\int \mathbf{E} \cdot d\mathbf{r}$$

Going in the other direction, from potential to field, requires that we use the concept of the gradient of the potential.

$$E = -\frac{dV}{dr}$$

For multiple dimensions

$$E_x = -\frac{\partial V}{\partial x}; E_y = -\frac{\partial V}{\partial y}; E_y = -\frac{\partial V}{\partial y};$$

where the field is

$$\mathbf{E} = E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}$$

Graphical problems might require the student to analyze a graph of potential as a function of position and take the negative of the slope at a given point to determine the field at that point. Some graphical problems are two dimensional. Equi-potential surfaces are drawn, and the student must identify the direction of the field at a given point by identifying the direction in which the potential is changing most rapidly, which occurs where the equi-potential lines are spaced most closely together. The field points in the direction of increasingly negative potential, as indicated by the negative sign in the derivative expression.

Conceptual Development Is Largely in the Links

In conclusion, the concept map presented in Figure 1 may seem on the surface to be merely a mnemonic for memorization of the equations contained within the circles, but careful and painstaking development of the links between the equations allows for plenty of discussion and conceptual development of the underlying physics. I have found that this approach provides me with an opportunity to emphasize to my students that the concepts of force, field, potential energy, and potential are tightly coupled. The parallelism in the mathematical development helps the students link the concepts together in a consistent and conceptually valid way.

Sample AP Questions for Physics B

1984 Physics B multiple-choice

15. The electron-volt is a measure of

(A) charge (B) energy (C) impulse (D) momentum (E) velocity

The answer is (B). The charge of a proton is e, and when the proton is moved through a potential difference of V, the resulting unit eV must be energy since $\Delta U = q\Delta V$. It should be pointed out that although this equation deals only with potential energy, if the electrostatic force represents the only (or net) force, kinetic energy will instead be created.

1988 Physics B multiple-choice

17. An electron is accelerated from rest for a time of 10^{-9} second by a uniform electric field that exerts a force of 8.0×10^{-15} Newton on the electron. What is the magnitude of the electric field?

(A) 8.0×10^{-24} N/C

(B) 9.1×10^{-22} N/C

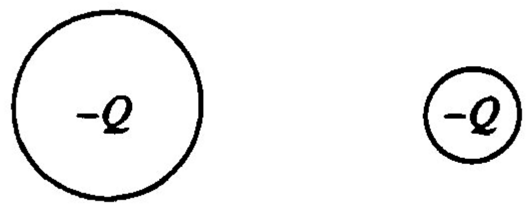
(C) 8.0×10^{-6} N/C

(D) 2.0×10^{-5} N/C

(E) 5.0×10^{4} N/C

The answer is (E). The charge of an electron is $-e$, which has a magnitude in the range of 10^{-19}C. From $\mathbf{F} = q\mathbf{E}$, it is apparent that for the force to have a magnitude in the range of 10^{-14}N, the field should be in the range of 10^{5}N/C. This problem is a great one for practicing scientific notation and estimation skills.

1993 Physics B multiple-choice



70. Two conducting spheres of different radii, as shown above, each have charge –Q. Which of the following occurs when the two spheres are connected with a conducting wire?

(A) No charge flows.

(B) Negative charge flows from the larger sphere to the smaller sphere until the electric field at the surface of each sphere is the same.

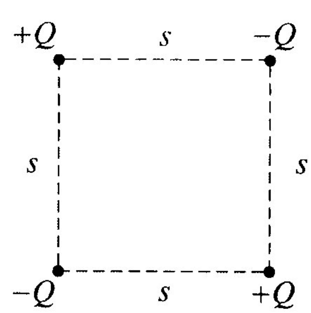
(C) Negative charge flows from the larger sphere to the smaller sphere until the electric potential of each sphere is the same.

(D) Negative charge flows from the smaller sphere to the larger sphere until the electric field at the surface of each sphere is the same.

(E) Negative charge flows from the smaller sphere to the larger sphere until the electric potential of each sphere is the same.

The answer is (E). Electrons will experience a force causing them to move along the wire provided there is an electric field on the wire and parallel to it, according to $F = qE$. Such an electric field will exist as long as there is a potential difference from one end of the wire to the other, as illustrated by $\Delta V = -E\Delta r$. So, negative charge will flow until the potential difference is eliminated. Since the smaller sphere has a higher negative potential according to $V = kq/r$, the direction of negative charge flow will be from the smaller to the larger sphere.

Sample Free Response Problem for Physics B



Arrangement 1

2001 B3.

Four charged particles are held fixed at the corners of a square of side s. All the charges have the same magnitude Q, but two are positive and two are negative. In Arrangement 1, shown above, charges of the same sign are at opposite corners. Express your answers to parts a. and b. in terms of the given quantities and fundamental constants.

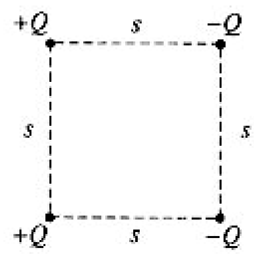
a. For Arrangement 1, determine the following.

i. The electrostatic potential at the center of the square

Answer: 0 (*scalar addition*)

ii. The magnitude of the electric field at the center of the square

Answer: 0 (*vector addition*)



Arrangement 2

The bottom two charged particles are now switched to form Arrangement 2, shown above, in which the positively charged particles are on the left and the negatively charged particles are on the right.

b. For Arrangement 2, determine the following.

i. The electrostatic potential at the center of the square

Answer: 0 (*scalar addition; rearrangement does not affect result*)

ii. The magnitude of the electric field at the center of the square

Answer:

$$E = \frac{\sqrt{2}Q}{\pi\varepsilon_0 s^2}$$

This answer is obtained by vector addition. Although the y-components of the field vectors will cancel each other, the x-components will not cancel and will produce a resultant field vector in the x-direction. For each charge, the field at the middle of the square will be

$$E_x = \frac{Q}{4\pi\varepsilon_0 r^2}\cos 45^\circ = \frac{Q}{4\pi\varepsilon_0 \frac{s^2}{2}}\frac{\sqrt{2}}{2}$$

Multiplication of E_x for one charge by the number of charges gives the answer.

c. In which of the two arrangements would more work be required to remove the particle at the upper right corner from its present position to a distance a long way away from the arrangement?

____*x*____ Arrangement 1 ____________ Arrangement 2

Justify your answer

Energy justification:

The location of the upper right (negative) charge is more favorable in arrangement 1 than it is in arrangement 2 due to the higher electric potential. Therefore, more work will be required to remove the upper right charge from arrangement 1.

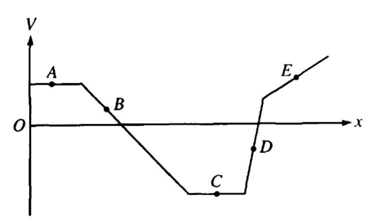
Force justification:

Attractive forces are more predominant in arrangement 1 than they are in arrangement 2, and overcoming these forces will require a larger applied force. Therefore, since a larger force is required to remove the upper right charge from arrangement 1, more work will be done since $W = Fd$.

Sample AP Questions for Physics C

Note: Although these are questions taken from Physics C examinations, in the opinion of the author these questions should also be able to be answered by Physics B students.

1998 Physics C multiple-choice



47. The graph above shows the electric potential V in a region of space as a function of position along the x-axis. At which point would a charged particle experience the force of greatest magnitude?

(A) A (B) B (C) C (D) D (E) E

The answer is (D). The field is strongest where the potential is changing most rapidly with respect to position (in Physics C language, the field is the negative gradient of the potential). On a potential vs position graph, the potential is changing most rapidly with respect to position where the slope is the steepest. Therefore, the larger the magnitude of the slope, the larger the magnitude of the field.

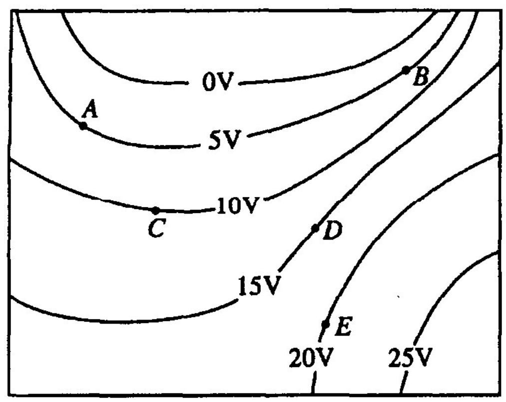
48. The work that must be done by an external agent to move a point charge of 2 mC from the origin to a point 3 m away is 5 J. What is the potential difference between the two points?

 (A) 4×10^{-4} V (B) 10^{-2} V (C) 2.5×10^{3} V

 (D) 2×10^{6} V (E) 6×10^{6} V

 The answer is (C). The work done by an external agent is equal to ΔU, and $\Delta U = q\Delta V$. The 3m distance is irrelevant.

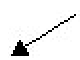
2004 Physics C multiple-choice

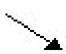
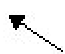


Questions 59–61

The diagram above shows equipotential lines produced by an unknown charge distribution. A, B, C, D, and E are points in the plane.

59. Which vector below best describes the direction of the electric field at point A?

 (A)  (B) 

 (C)  (D) 

 (E) None of these; the field is zero.

 The answer is (A). The field is the negative gradient of the potential, and points "downhill" toward lower potential. There is always a right angle between an equipotential line and a field vector.

60. At which point does the electric field have the greatest magnitude?

 (A) A (B) B (C) C (D) D (E) E

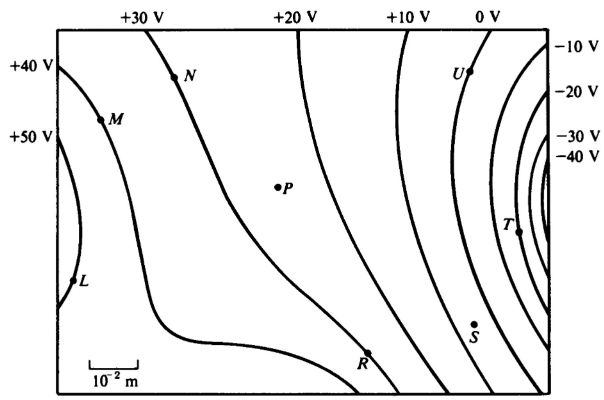
 The answer is (B). Because $E = -\Delta V/\Delta R$, when the spacing between two equipotential lines is smallest, E must be biggest.

61. How much net work must be done by an external force to move a $-1\ \Delta C$ point charge from rest at point C to rest at point E ?

 (A) $-20\ \mu J$ (B) $-10\ \mu J$ (C) $10\ \mu J$ (D) $20\ \mu J$ (E) $30\ \mu J$

The answer is (B). Because $U = q\Delta V$ and $W_{ext} = \Delta U$, then $W_{ext} = q\Delta V$. The negative charge will be moving to a more favorable location at higher potential, so the external work will be negative. The potential change is +10V, so the external work done will be −10μJ.

Sample Free Response Problem for Physics C



1986 E1.

Three point charges produce the electric equipotential lines shown on the diagram above.

a. Draw arrows at points L, N, and U on the diagram to indicate the direction of the electric field at these points.

The field vectors are at right angles to the potential lines, and point toward the negative potentials. This is a graphical representation of a gradient in a two-dimensional system.

b. At which of the lettered points is the electric field E greatest in magnitude? Explain your reasoning.

The answer is T, since the equipotential lines are most closely spaced near T. Again, this is a graphical representation of the gradient.

c. Compute an approximate value for the magnitude of the electric field E at point P.

Using $E = -\Delta V/\Delta x$, the magnitude of the electric field can be estimated to be 500 V/m. The potential difference between adjacent equipotential lines is 10 Volts, and the distance between these lines is estimated to be 0.02m.

d. Compute an approximate value for the potential difference, $V_M - V_S$, between points M and S.

Answer: 35 Volts.

e. Determine the work done by the field if a charge of $+5 \times 10^{-12}$ coulomb is moved from point M to point R.

The answer is 5×10^{-11}J. $\Delta U = q\Delta V$, which is -5×10^{-11}J, and the work done by the conservative electric field force is the negative of the potential energy change.

f. If the charge of $+5 \times 10^{-12}$ coulomb were moved from point M first to point S, and then to point R, would the answer to (e) be different, and if so, how?

It would be the same, since work done by a conservative force does not depend upon path.